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ACCELERATED FATIGUE TEST RATIONALE.

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ACCELERATED FATIGUE

TEST RATIONALE

ACCELERATED FATIGUE TEST

RATIONALE

INTRODUCTION

It is typically of great interest and practical importance to accelerate a vibration or thermal cycling qualification or acceptance test in the laboratory from the actual service conditions. The test duration is compressed by a relatively large factor (e.g. 1000) with an attendant increase in applied vibration or temperature limit levels. The relationship between test duration compression and level increase that has been used by the industry is as follows:

$$\frac{L_2}{L_1} = \left(\frac{T_1}{T_2}\right)^{1/\alpha} \tag{1}$$

where T_1 = service life

T_2 = test duration

L_1 = service level

L_2 = test level

α = constant

Various industry groups assign various values to α for a variety of reasons and conservativeness. Values of α between 4 to 9 have been used. There is much disagreement between groups on the value assigned to α. As an example for $\alpha = 4$ and a 1000 hour service life compressed into a one hour test

$$\frac{L_2}{L_1} = (1000)^{1/4} = 5.6$$

Thus, the "laboratory" test level should be 5.6 larger than the service level in order that the cumulative fatigue damage is the same in both cases. Fatigue damage is not to be misinterpreted as fatigue failure (i.e. fracture).

Fatigue failures may or may not occur depending upon the magnitude of the cumulated damage. The previous equation can be interpreted to mean that the potential for fracture will be the same for a service duration T_1 at level L_1 as for a test duration T_2 at Level L_2.

This paper shows a logical basis for selecting the form of the relationship and assigning values to the parameters. For Fatigue (i.e. no initial flaws) the form of the correct expression is as previously shown. The introduction of Fracture Mechanics effects (i.e. initial flaws) results in a different form of the expression that relates levels, durations and different parameter values. Non-linear damping effects are also included. Conditions of similitude between the service and accelerated test environments are identified. Such conditions must be ensured if the developed equations are to be applied appropriately and accurately (e.g. no new failure mechanism should be introduced at the accelerated test level).

Techniques are described that allow compensating during the accelerated test for differences in response stress spectra or distribution of stress peaks (clipping) between the service and laboratory environments.

APPROACH SUMMARY

$D_1 = \frac{N_1}{N_{f_1}}$ (2) ; $D_2 = \frac{N_2}{N_{f_2}}$ (3)

D_1 = damage cumulated in service environment

N_1 = number of applied stress cycles at stress level ΔS_1 , σ_1

N_{f_1} = number of stress cycles to failure at stress level 1

D_2 = damage in accelerated environment

N_2 = number of applied stress cycles at stress level ΔS_2 , σ_2

N_{f_2} = number of stress cycles to failure at stress level 2

ΔS = sinusoidal stress range, peak-peak stress

σ = random stress rms level

Fatigue failure (fracture) occurs when D_1 or D_2 = 1.

$N_1 = f_1 T_1$; $N_2 = f_2 T_2$

f_1 = frequency of stress cycles at level ΔS_1 , σ_1

T_1 = time or duration of applied stress at level ΔS_1 , σ_1

f_2 = frequency of stress cycles at level ΔS_2 , σ_2

T_2 = time or duration of applied stress at level ΔS_2 , σ_2

Time compression ratio $= \frac{T_1}{T_2} = \frac{N_1}{N_2}$ for $f_1 = f_2$ (4)

$N_2 < N_1$

Accelerated test level $= \frac{\Delta S_2}{\Delta S_1}$, $\frac{\sigma_2}{\sigma_1}$ (5)

Fatigue failures may or may not occur at either the environmental or accelerated test levels. Whether they occur is not of interest for this analysis. It is of interest, however, that the accumulated fatigue damage be the same for both conditions (i.e. $D_1 = D_2$). For example, if 60% of life (no fatigue failure) is accumulated after N_1 cycles at the service environmental stress level ΔS_1 or σ_1, it is desired to find the accelerated test stress level ΔS_2 or σ_2 that will correspondingly accumulate the same 60% of life after only N_2 cycles (i.e. $D_1 = D_2 = 0.60$). Also if 120% of life (fatigue failure) is accumulated at the service level, it is desired to accumulate 120% of life at the accelerated test level (i.e. $D_1 = D_2 = 1.20$).

Thus

$$D_1 = D_2 \tag{6}$$

$$\frac{N_1}{N_{f_1}} = \frac{N_2}{N_{f_2}} \tag{7}$$

or

$$\frac{N_2}{N_1} = \frac{N_{f_2}}{N_{f_1}} \tag{8}$$

The fatigue and test parameters are related directly to N_{f_1} and N_{f_2}, not to N_1 and N_2. The analysis that follows, therefore, will be in terms of N_{f_2}/N_{f_1}. This ratio is the same as N_2/N_1. Expressions in terms of fatigue failure parameters are a mathematical and engineering necessity but should not be misinterpreted to mean that fatigue failures will occur at either level.

The ratio N_{f_2}/N_{f_1} (i.e. the ratio N_2/N_1) will be a function of the corresponding stress levels. This equation will then be rearranged so that the ratio $\Delta S_2/\Delta S_1$ or σ_2/σ_1 can be solved for in terms of the "time" compression factor N_1/N_2.

The fatigue process can be characterized by crack initiation, stable crack propagation and fracture (i.e. unstable crack growth when the crack size equals the critical crack size value). Each of these individual processes is directly a function of the stress or strain level and the number of applied stress cycles. The fatigue process is only indirectly related to the input vibration acceleration level to the structural elements in, say, an electronic "black box" or to the mission or test duration.

Consequently relationships have been established that relate stress levels to input vibration acceleration levels and number of stress cycles to mission or test duration. This allows the input vibration acceleration levels $\ddot{x}_2/\ddot{x}_1$ to be functionally related to the time compression factor T_1/T_2.

Linear and non-linear dependence of stress upon input vibration level is included in the term η. $\eta = 1$ corresponds to a linear relationship. It is shown that $\eta = 0.714$ for sinusoidal vibration inputs and $\eta = 0.833$ for random vibration inputs where the predominant damping mechanism is internal stress-strain hysteresis damping. For $\eta < 1$ the effective damping at resonance increases more than proportionally with an increase in input vibration level. $\eta > 1$ applies to cases where the predominant damping mechanism is Coulomb friction or where the effective spring stiffness increases with input vibration level, as examples.

Fracture Mechanics effects (i.e. initial cracks or flaws) have been distinguished from the usual Fatigue effects (i.e. no initial flaws). The major difference is in the number of stress cycles (hence, time) required to initiate cracks for Fatigue. Cracks (either actual or postulated) already exist for Fracture Mechanics.

The solution of the accelerated stress level ratio in terms of the stress cycle "compression ratio" and the accelerated vibration input level ratio in terms of the time compression ratio involves solving transcendental functions. Computer program in Basic Language that accomplish that task are included.

CONDITIONS OF SIMILITUDE

Certain conditions of similitude must be imposed upon the service and laboratory accelerated test environments if the developed mathematical relationships are to be appropriately and accurately applied. The fundamental hypothesis is that the damage states and damage rates must be the same for both environments. Specifically the states of stress (torsion, bending, axial), the corresponding strengths, the resonant mode shapes, the internal response stress spectrum shapes, the stress peak distribution, and the type and location of failure mechanisms must be the same for both environments.

Differences in temperature, rate of stressing, corrosive environments, and other environmental effects (e.g. "purple plague" that can result from combined high temperature and humidity) between the service and test conditions may violate conditions of similitude for some structural elements. Violation will occur if the above factors are sufficient to alter the material's fatigue strength (i.e. fatigue curve parameters) between the two environments.

Threshold sensitive or other non-linear response effects in general tend to violate conditions of similitude. In some cases lack of similitude can be quantitatively compensated for. Several examples are included in Appendix A.

The condition that the shape of the vibration acceleration input spectra must be the same for both environments has purposely been omitted from the previously listed conditions. This is because the fatigue damage state and rate are only indirectly related to the input acceleration spectrum.

They are directly related to the response stress spectrum at the location where damage is accumulating. Stress is herein defined as the internal force per unit area in a material that results from the application of an external load. The input vibration acceleration to a "black box" is defined as the kinematic motion response at the load transfer path input location that results from applied vibratory loads to the black box and adjacent structural members. The input vibration acceleration is not an "applied stress" using the above definitions.

FATIGUE

SINE VIBRATION

From reference [1] a material's sine fatigue curve is

$$S = \frac{\Delta S}{2} = \bar{A}\, N^{-1/\beta} \tag{9}$$

$$\frac{S_2}{S_1} = \left(\frac{N_1}{N_2}\right)^{1/\beta} \quad ; \quad \frac{N_1}{N_2} = \frac{f_1 T_1}{f_2 T_2} = \frac{T_1}{T_2}$$

$$S = C_2\, \ddot{x}^{\eta} \tag{10}$$

$$\left(\frac{\ddot{x}_2}{\ddot{x}_1}\right)^{\eta} = \left(\frac{T_1}{T_2}\right)^{1/\beta} \tag{11}$$

or

$$\ddot{x}_2 = \ddot{x}_1 \left(\frac{T_1}{T_2}\right)^{1/\eta\beta} \quad \text{g's} \tag{12}$$

η = 1 for linear damping (refer to Appendix B)

$\ddot{x}_2$ = represents the accelerated test input acceleration level (g's)

$\ddot{x}_1$ = service vibe input acceleration level (g's)

T_1, T_2 = corresponding durations

FATIGUE

RANDOM VIBRATION

From reference [1] the random fatigue curve equation is

$$\sigma = \bar{C}\, N^{-1/\beta} \tag{13}$$

where

σ = rms stress (KSI)

$\bar{C}$ = constant (KSI)

N = average cycles to failure

β = slope parameter of the material's sine fatigue curve

$$N = fT \tag{14}$$

where

f = center frequency of narrow-band response (Hz)

T = duration

In a fashion similar for the sinusoidal case the accelerated test level is

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{N_1}{N_2}\right)^{1/\beta} = \left(\frac{T_1}{T_2}\right)^{1/\beta} \tag{15}$$

$$\sigma = C4\, \ddot{x}^{n} \tag{16}$$

$$\left(\frac{\ddot{x}_2}{\ddot{x}_1}\right)^{\eta} = \left(\frac{T_1}{T_2}\right)^{1/\beta}$$

or

$$\ddot{x}_2 = \ddot{x}_1 \left(\frac{T_1}{T_2}\right)^{1/\eta\beta} \qquad (17)$$

η = 1 for linear damping

$\ddot{x}_1$ = service vibration input acceleration rms level

T_1 = duration of service vibe

$\ddot{x}_2$ = accelerated test vibe input acceleration rms level

T_2 = duration of accelerated test

The above equations assume that the ratio of the input Power Spectral Density (PSD) W_o in the vicinity of resonance is

$$\frac{W_{o_2}}{W_{o_1}} = \left(\frac{\ddot{x}_2}{\ddot{x}_1}\right)^2 \qquad (18)$$

Otherwise, the conditions of similitude will be violated.

LOW CYCLE FATIGUE

Mechanical loads or deformations that are of sufficient magnitude to stress the material into the plastic (i.e. inelastic) region of its stress-strain curve are associated with short fatigue lifes. This is often referred to as low cycle fatigue. Fatigue lifes typically extend up to approximately 10^4 stress cycles. The high cycle fatigue (i.e. elastic stress-strain) typically extends beyond 10^4 cycles.

The Coffin-Manson low cycle fatigue expression relates the applied plastic strain amplitude, the material's ductility and cycles to failure for cyclically induced strains in mechanical systems. Static stresses do not affect fatigue life in the low cycle region and therefore can be ignored for this analysis.

$$\frac{\Delta\varepsilon}{2} = \varepsilon'_f \ (2\,N_f)^{-1/\beta} \tag{19}$$

$\frac{\Delta\varepsilon}{2}$ = applied plastic strain amplitude (in/in)

ε'_f = fatigue ductility coefficient (in/in)

N_f = cycles to failure

β = slope parameter ≈ 2 for most structural materials

The above strains are "true" strains which include changes in the strained specimen's cross-sectional area under load as compared to "engineering" strains which are based upon the specimen's elongation relative to its original length. Until specimen necking occurs:

$$\varepsilon_{true} = \ell n \left(1 + \varepsilon_{eng'g}\right) \tag{20}$$

"Engineering strain is usually more convenient to use than "true" strain. The Coffin-Manson can be modified [2] to give

$$\varepsilon = \varepsilon_u \left(2 N_f\right)^{-1/\beta} \tag{21}$$

ε = applied "engineering" plastic strain amplitude (in/in)

ε_u = material ductility; ultimate percent elongation (in/in)

N_f and β are the same as before

TABLE I CORRESPONDING STRAIN PARAMETERS

Eng'g	True
ε	$\Delta\varepsilon/2$
ε_u	ε'_f

Using true strains the service and accelerated test levels can be related as follows:

$$\frac{\Delta\varepsilon_2}{\Delta\varepsilon_1} = \left(\frac{N_1}{N_2}\right)^{1/\beta} \tag{22}$$

where the subscript 1 applies to the service environment and the subscript 2 applies to the accelerated test environment.

Engineering strains can be substituted into the above expression. The results will then be accurate if no necking of the structural element occurs. The results will be conservative if necking does occur, because the material is actually more ductile than given credit for.

The above expression applies to all forms of cyclic strain. Strains resulting from temperature cycling is typical.

As an example consider a glass epoxy multi-layer board (MLB) that is to be subjected to temperature cycling. The differential expansion rate in a direction perpendicular to the plane of the board between the epoxy and the electrodeposited copper plated-through-holes (PTH) is non-linearly related to temperature. Assume that the service temperature cycle limits of 0°C to +95°C produces an applied strain amplitude of 8.45×10^{-4} in/in in the middle region of the PTH's where there is a potential for circumferential cracking. Assume that the quantity of service temperatures cycles is 7000 cycles. It is desired to find the accelerated test temperature range to cumulate the same fatigue damage in only 2550 cycles.

$$\varepsilon_2 = \varepsilon_1 \left(\frac{N_1}{N_2}\right)^{0.5} = 8.45 \times 10^{-4} \left(\frac{7000}{2550}\right)^{0.5}$$

$$\varepsilon_2 = 0.0014 \text{ in/in}$$

Measured strain amplitude versus temperature limits indicates that 0.0014 in/in corresponds to limits of -65°C to +125°C for the particular MLB. Care must be taken to ensure that a new failure mechanism is not introduced (e.g. PTH corner cracks).

Care must be exercised when considering materials whose fatigue properties are rate or test temperature sensitive [3]. Consider 63 - 37 Tin-Lead Solder plastically stressed in reversed shear:

TABLE II SOLDER SHEAR FATIGUE PARAMETERS

TEST TEMPERATURE (C°)	SHEAR STRAIN RATE (cycles per minute)	β	FATIGUE EXPRESSION (in/in)
25	1/15	2.63	$\Delta\varepsilon = 0.531\ N^{-0.381}$
25	5	3.31	$\Delta\varepsilon = 0.560\ N^{-0.302}$
100	5	2.87	$\Delta\varepsilon = 0.488\ N^{-.0348}$

FRACTURE MECHANICS EFFECTS

The primary fracture mechanic's effects are those due to initial cracks (flaws) that are either actual or hypothesized. Such cracks reduce fatigue life. They either exist in the structural material as metallurgical inclusion or dislocations or are introduced during manufacturing fabrication and assembly operations. They can also be created by temporary overloads into the plastic stress regions.

FATIGUE CURVES

From typical fatigue curve data (e.g. reference [4] or reference [5])where any initial flaw sizes are approximately zero the usual form of the fatigue curve is

$$\frac{\Delta S}{2} = \bar{A}\, N^{-1/\beta} \;.$$

For 7075-T6 Aluminum Alloy

$$\frac{\Delta S}{2} = 180\ N^{-0.104} \qquad \text{KSI}$$

From Fracture Mechanics (See Appendix C)

$$N_f = \frac{1.698 \times 10^7}{\Delta S^4}\left[\frac{1}{a_i} - 7.83 \times 10^{-3}\ \Delta S^2\right]$$

The above N_f expression applies for a particular geometry.

Both types of fatigue curves are plotted in figure 1 . It can be seen that even a small value of a_i reduces fatigue life. Further observations are:

a) The slope parameter of the usual fatigue curve ($a_i = 0$) is $\beta = 9.65$ for 7075-T6.

b) The slope parameter of the fracture mechanics fatigue curve ($a_i = 0$) is Θ. $\Theta = 4$ for 7075-T6.

NOTE: $\Theta \neq \beta$

c) For large N_f (7075-T6)

$$\frac{\Delta S}{2} \approx \left[\frac{1.06 \times 10^6}{a_i \ N_f} \right]^{0.25}$$

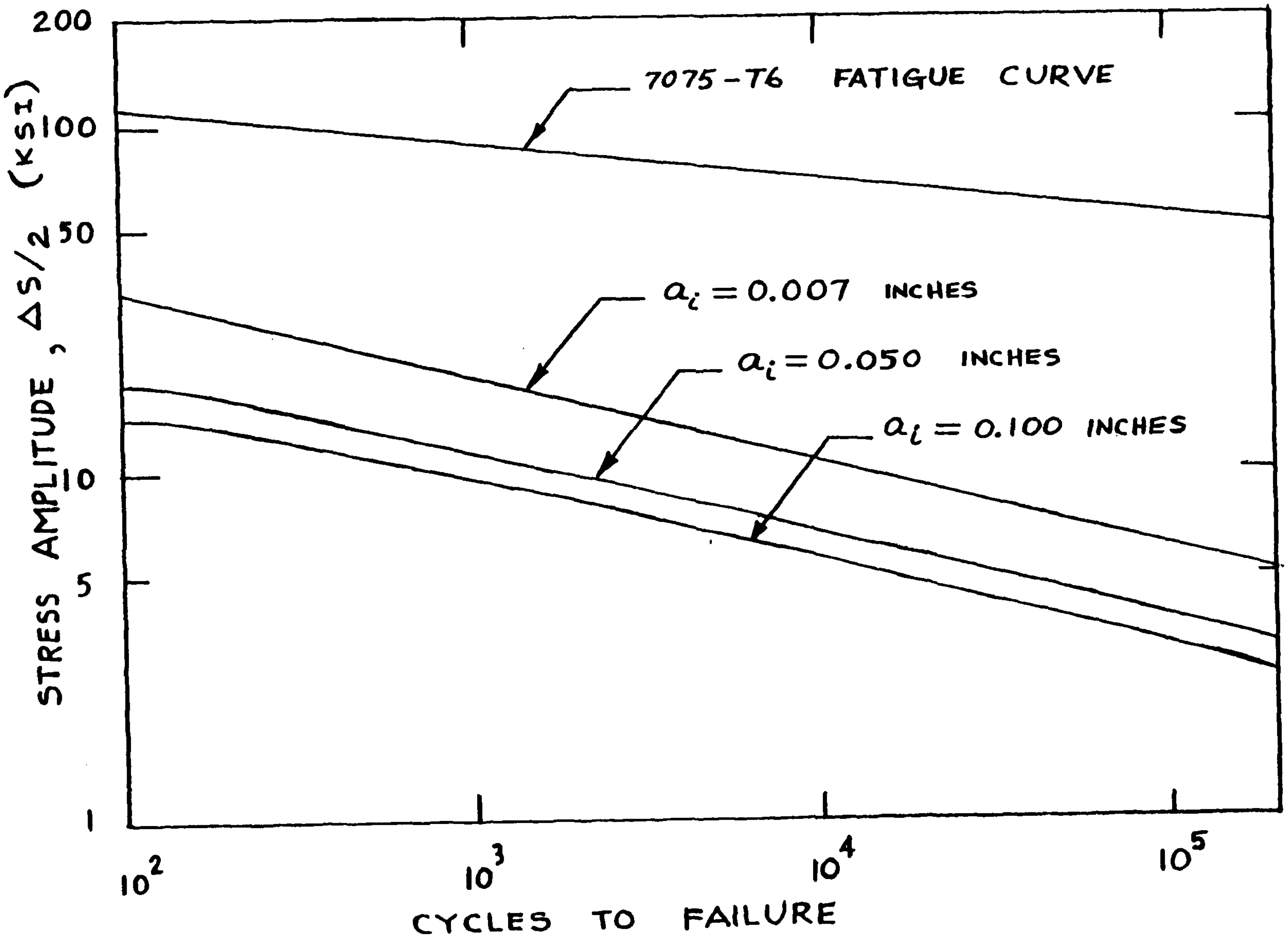

FIGURE 1 FAILURE CURVES - VARIABLE a_i

ACCELERATED SINE STRESS LEVEL

Appendix D′ (FRACTURE MECHANICS ACCELERATED SINUSOIDAL TEST STRESS LEVEL) shows the derivation of the accelerated sine test stress level ΔS_2 given the environmental stress level ΔS_1 and the corresponding test duration (i.e. cycles) N_1 and N_2. The transcendental function cannot be normalized (e.g. $\Delta S_2/\Delta S_1$ versus N_2/N_1) because of the inherent non-linearities in the fracture mechanics correction factor X. Thus, ΔS_1 and N_1 must be assigned specific values.

The Basic Language computer program PL-2 solves for ΔS_2, the accelerated stress level. The inputs are N_1, N_2, Θ, Y, a_i, ΔK_c, ΔS_1. The listing shows typical parameter values for 7075-T6 Aluminum Alloy. It should be noted that ΔK_c was chosen to be 20 KSI$\sqrt{\text{IN.}}$ [5] This value was the lowest (hence, the most conservative) value published in the literature. Much higher values; unfortunately, have also been published. Therefore, care must be exercised in using published data.

It is recommended that the most reliable data is that obtained using test method ASTM E647-78T, "Tentative Test Method for Constant Load Amplitude Fatigue Crack Growth Rates Above 10^{-8} m/cycle." This method uses as an applicable document ASTM 399 "Test for Plane-Strain Fracture Toughness of Metallic Materials." Reference [6] uses this test method.

Several runs using PL-2 are included for 7075-T6. See Groups I - III.

$N_1 = 10^6$ cycles ; $\Delta S_1 = 10$ KSI

TABLE III GROUP N_2 VALUES

GROUP	N_2 (CYCLES)	N_1/N_2
I	10^2	10^4
II	10^3	10^3
III	10^4	10^2

TABLE IV GROUP ACCELERATED STRESS LEVELS

a_i (INCHES)	I ΔS_2 (KSI)	II ΔS_2 (KSI)	III ΔS_2 (KSI)
0.007	87.4	53.9	31.2
0.050	49.1	42.2	28.9
0.100	35.5	33.6	26.5
0.200	-	24.8	-
0.500	-	15.9	-
1.000	-	11.3*	-

* The PL-2 execution results listed "NO SOLUTION". This occurs when $\Delta S_2 \approx X(1)$ MAX. In the above case X(1) MAX = 11.3 KSI.

To convert to from stress-cycles to input acceleration-time parameters the following relationships apply for the example in Appendix E .

$$\frac{\Delta S}{2} = C_2 \ddot{x}_S^{\,n_S} \tag{23}$$

$n_S = 1$; $f_n = 50$ Hz (resonance dwell)

$$T_S = \frac{N_S}{3000} \text{ minutes} \tag{24}$$

N_S = cycles

$$\ddot{x}_S = \frac{\Delta S/2}{38.1} = \frac{\Delta S}{76.2} \quad \text{g's} \tag{25}$$

Figure 2 shows the converted results. The accelerated levels are not sensitive to a_I values for small time compression ratios but are sensitive for large time compression ratios.

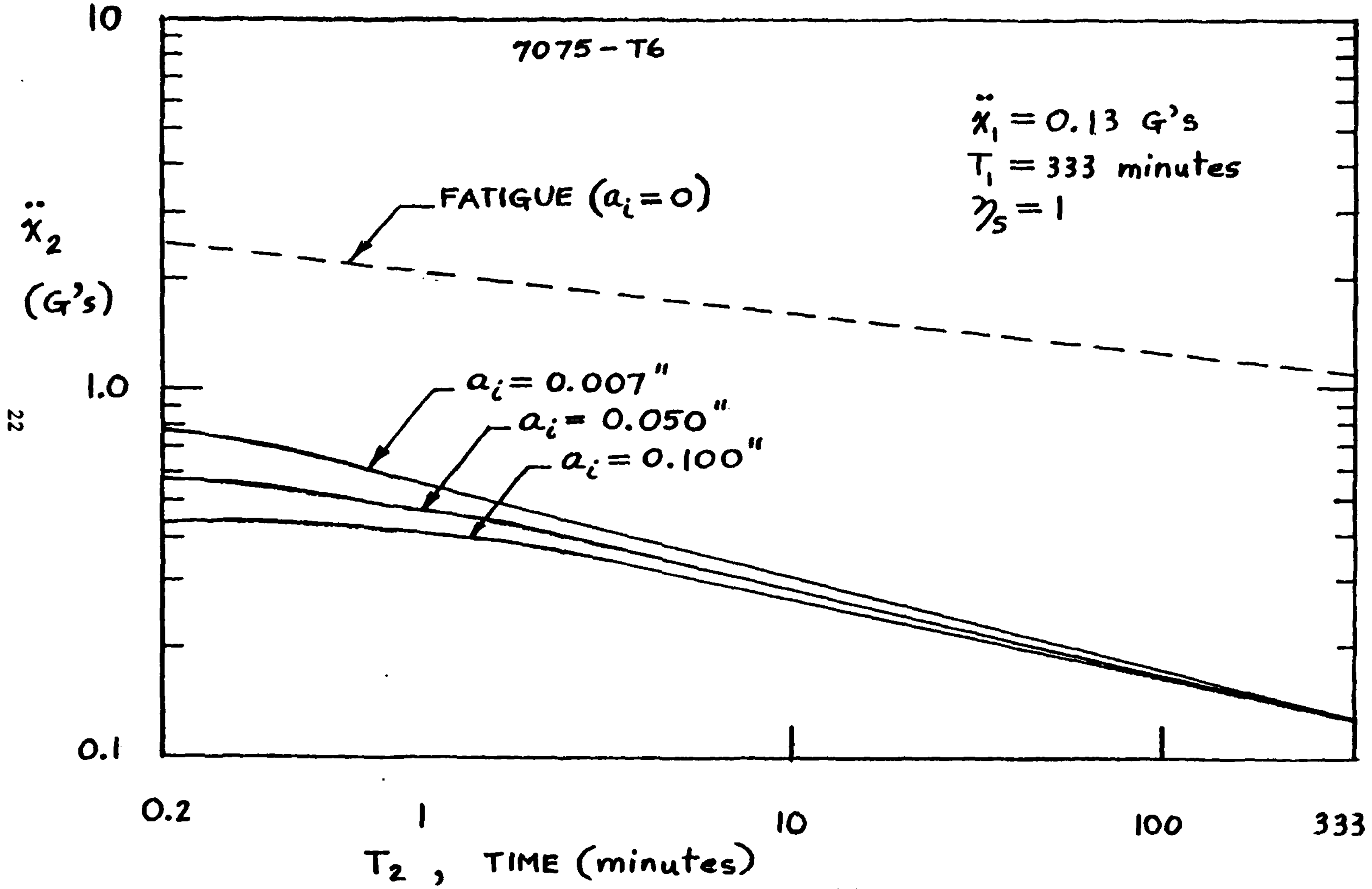

FIGURE 2 ACCELERATED SINE VIBE TEST LEVEL FOR $a_i = 0.007, 0.05, 0.1$ INCHES

ACCELERATED SINUSOIDAL STRESS LEVEL COMPUTATION PROGRAM (BASIC LANGUAGE)

```
1Ø REM N1= CYCLES-TO-FAILURE AT THE IN-FLIGHT
2Ø REM     ENVIRONMENTAL STRESS LEVEL
3Ø REM N2= CYCLES-TO-FAILURE AT THE LABORATORY
4Ø REM     ACCELERATED STRESS LEVEL
5Ø REM T1= THETA: CRACK GROWTH PARAMETER
6Ø REM  Y= FRACTURE MECHANIC GEOMETRICAL PARAMETER
7Ø REM A1= INITIAL CRACK LENGTH (INCHES)
8Ø REM K1= FRACTURE TOUGHNESS (KSI-SQ ROOT INCHES)
9Ø REM S1= IN-FLIGHT ENV'L STRESS LEVEL (KSI)
1ØØ REM R1=TIME COMPRESSION RATIO
11Ø REM H3=MAXIMUM POSSIBLE ACCEL'D STRESS LEVEL(KSI)
12Ø REM X(1)= INITIAL VALUE OF ACC'D LEVEL
13Ø REM R2=STRESS LEVEL RATIO
14Ø REM S2=ACCELERATED STRESS LEVEL (KSI)
15Ø N1=1E6
16Ø N2=1E3
17Ø T1=4
18Ø Y=1.77
19Ø A1=.ØØ7
2ØØ K1=2Ø
21Ø S1=1Ø
22Ø T2=1/T1
23Ø R1=N1/N2
24Ø E1=(T1-2)/2
25Ø E2=T1-2
26Ø E3=T1-3
27Ø E4=1/E2
28Ø V1=S1*(R1**T2)
29Ø LØ=Ø
3ØØ H1=((A1*Y**2)/(K1**2))**E1
31Ø H2=1/H1
32Ø H3=H2**E4
33Ø PRINT "X(1) MAX=";H3
34Ø PRINT
35Ø L1=1-(H1*S1**E2)
36Ø DIM P(2ØØØ),Q(2ØØØ)
```

ACCELERATED SINUSOIDAL STRESS
LEVEL COMPUTATION PROGRAM
(BASIC LANGUAGE)
CONTINUED

```
370 FOR J=1 TO 999
380 P(J)=H3*(J/1000)
390 B5=((1/L1)-((H1/L1)*P(J)**E2))
400 Q(J)=P(J)-V1*(B5**T2)
410 NEXT J
420 FOR J=2 TO 999
430 R5=Q(J)/Q(J-1)
440 IF R5>0 THEN 460
450 GO TO 500
460 NEXT J
470 PRINT "NO SIGN CHANGE: NO SOLUTION"
480 L0=1
490 GO TO 540
500 M1=Q(J)*P(J-1)+P(J)*ABS(Q(J-1))
510 M2=Q(J)+ABS(Q(J-1))
520 S2=M1/M2
530 PRINT
540 PRINT "N1,N2,T1,Y:"
550 PRINT N1,N2,T1,Y
560 PRINT
570 PRINT "A1,K1,S1:"
580 PRINT A1,K1,S1
590 PRINT
600 IF L0=1 GO TO 690
610 R2=S2/S1
620 X0=R1/(R2**T1)
630 PRINT "TEST TIME COMPRESSION FACTOR=";R1
640 PRINT "CORRECTION FACTOR,X=";X0
650 PRINT
660 PRINT "STRESS LEVEL RATIO";R2
670 PRINT
680 PRINT "ACCELERATED STRESS LEVEL=";S2
690 END
```

ACCELERATED SINUSOIDAL VIBE TEST ACCELERATION LEVEL

In the previous section the accelerated stress ΔS_2 was calculated given ΔS_1, N_1 and N_2. The corresponding sine acceleration - time parameters can be calculated from the stress - cycles parameters. Use PL-2 to obtain the stress-cycle parameters. Then compute the acceleration-time parameters using the following equations.

$$\ddot{x}_{1_{sine}} = \left(\frac{\Delta S_1}{2C_2}\right)^{1/\eta_s} \tag{26}$$

$$\ddot{x}_{2_{sine}} = \left(\frac{\Delta S_2}{2C_2}\right)^{1/\eta_s} \tag{27}$$

$$T_1 = N_1/f_n \tag{28}$$

$$T_2 = N_2/f_n \tag{29}$$

where f_n = resonant frequency

See Appendix E (SINE-RANDOM EQUIVALENCE) for an example of C_2 .

ACCELERATED RANDOM VIBRATION TEST LEVEL

Appendix F (Derivation of Accelerated Random Vibe Test Acceleration Level) shows the derivation of the accelerated random vibe test acceleration input level $\ddot{x}_2$ (g rms) given the environmental input acceleration level $\ddot{x}_1$ (g rms) and the corresponding test durations T_1 and T_2 . The expression is:

$$\ddot{x}_2 - \ddot{x}_1 \left(\frac{T_1}{T_2}\right)^{\frac{1}{\eta\ \theta}} \left(\frac{1}{X}\right)^{\frac{1}{\eta\ \theta}} = 0 \qquad (30)$$

This transcendental function cannot be normalized in closed form in terms of $\ddot{x}_2/\ddot{x}_1$ versus T_1/T_2 because of the inherent non-linearities in the fracture mechanics correction factor X . Thus $\ddot{x}_1$ and T_1 must be assigned specific values.

The Basic Language Computer program listing PL-3 solves the previous transcendental function. The program inputs are the test durations T1 and T2, θ , η , a_i , $\ddot{x}_1$, C4 , $\bar{A}$, $\bar{C}$, ΔK_c and Y . See PL-3 for further details. The program output is the accelerated random vibe test input rms acceleration value, $\ddot{x}_2$.

Figure 3 shows a plot of $\ddot{x}_2$ versus T2 for several values of a_i . $\ddot{x}_1$ = 1 g rms , T1 = 1000 hours and η = 0.833 . These curves are almost straight lines on a log-log plot. Increasing values of a_i from 0.007 inch to 0.100 inch does not greatly alter the curves for the T_2 values shown. For lower values of T_2 the curves deviate more from each other similar to those of figure 2 .

Figure 4 has the same parameter values as figure 3 except $\eta = 1$. The results are similar. Figure 5 has $\eta = 1.2$. Again the results are similar. The curves of all three figures have slopes on the log-log plots as follows:

$$\text{slope} \approx \frac{1}{\eta\theta}$$

Thus, it might be expected that the results should be sensitive to η values. Figure 6 confirms this expectation. Since θ is a material property, the results are also sensitive to the material. Table V shows several material θ values.

TABLE V θ Values

MATERIAL	θ
A-286	3.24
A 471 CL 4	1.4
Cr-Mo-V	4.09
4340	4.65
7075-T6	4.00

It will be noted that for all fracture mechanic's examples in this study figure 7 will apply for simplicity.

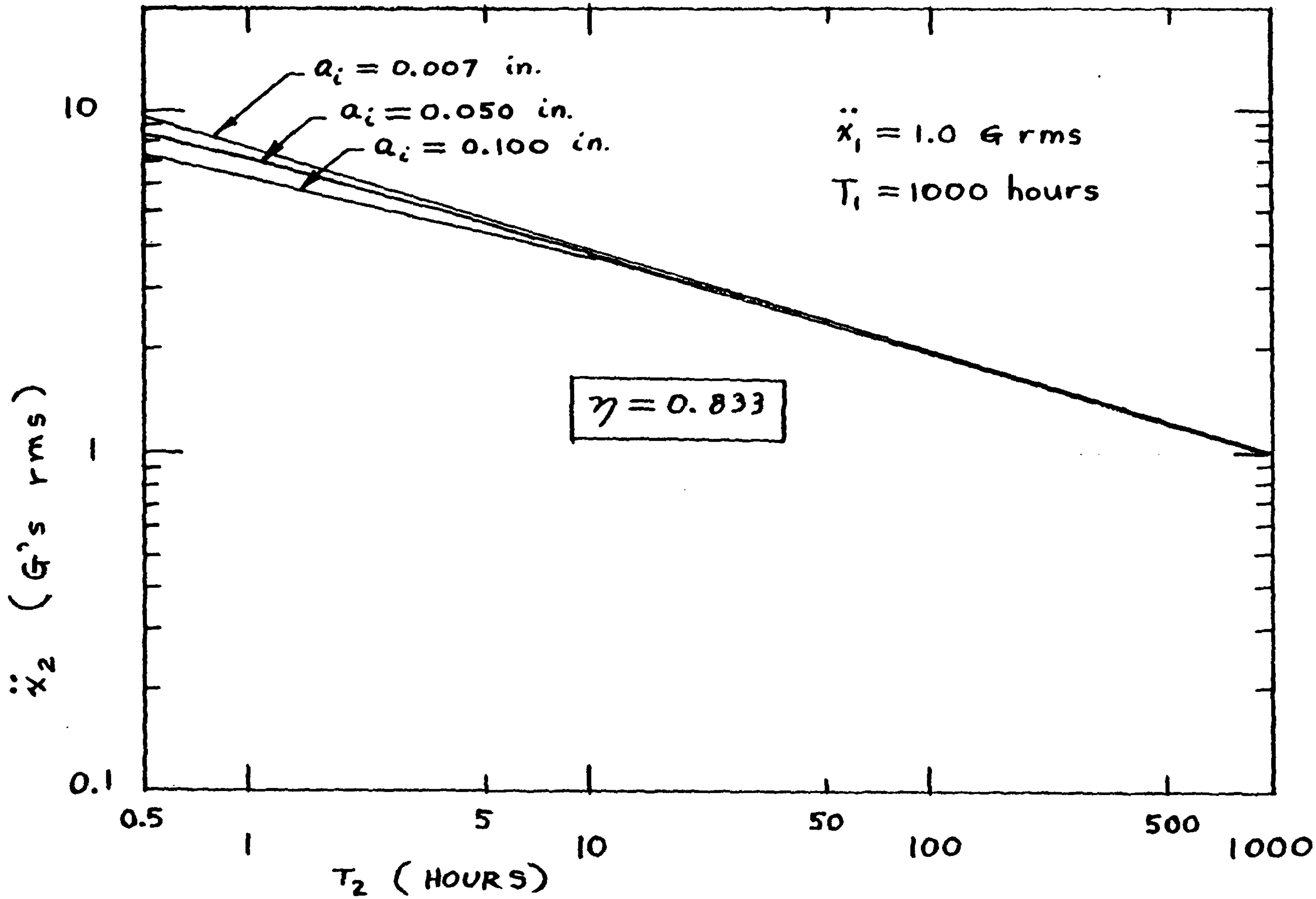

FIGURE 3 ACCELERATED RANDOM INPUT ($\eta = 0.833$)

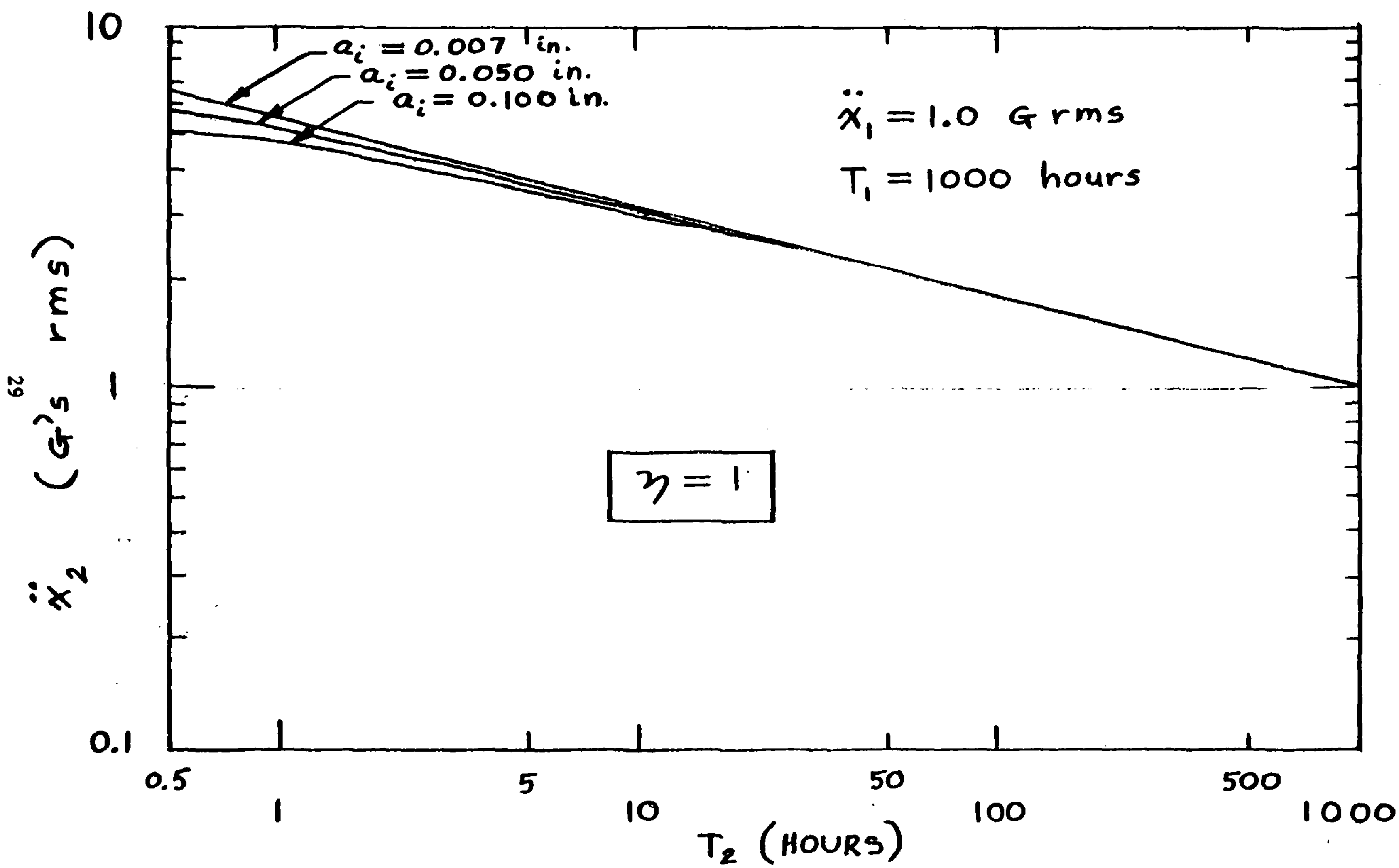

FIGURE 4 ACCELERATED RANDOM INPUT ($\eta = 1$)

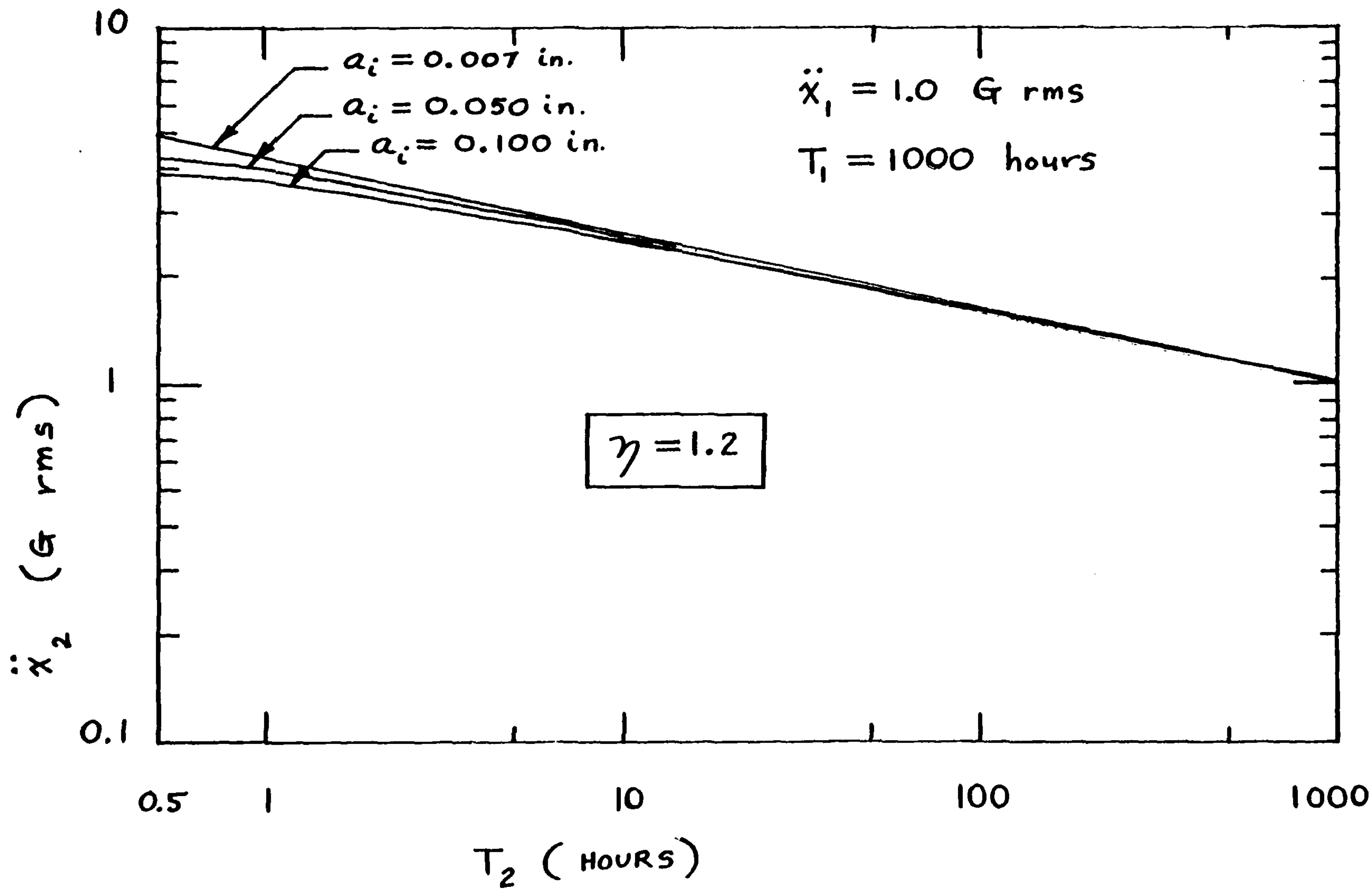

FIGURE 5 ACCELERATED RANDOM INPUT ($\eta = 1.2$)

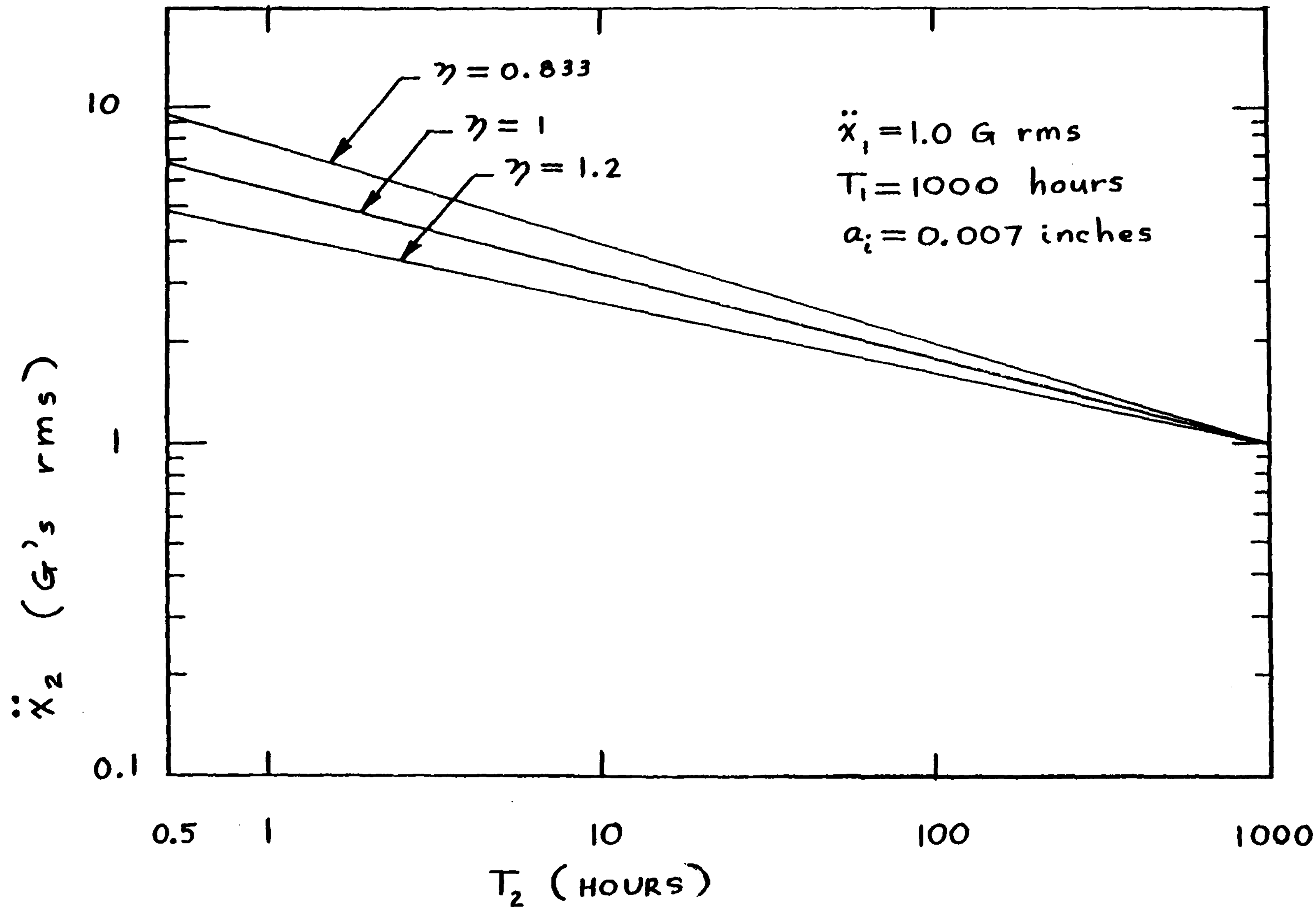

FIGURE 6 ACCELERATED RANDOM INPUT (η VARIABLE)

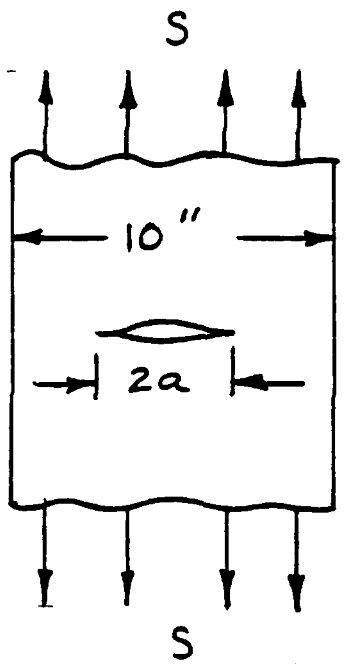

FIGURE 7 CENTER CRACKED STRIP LOADED IN TENSION

ACCELERATED RANDOM TEST
LEVEL COMPUTATION PROGRAM
(BASIC LANGUAGE)

```
10 REM T1=TEST DURATION AT LEVEL 1 (HRS)
20 REM T2=TEST DURATION AT LEVEL 2 (HRS)
30 REM TØ=THETA
40 REM EØ=ETA
50 REM A1=INITIAL CRACK LENGTH (INCHES)
60 REM G1=ACCELERATION LEVEL 1 (G'S RMS)
70 REM G2=ACCELERATION LEVEL 2 (G'S RMS)
80 REM C4=RMS STRESS PER G RMS (KSI/G)
90 REM A9=AVG VALUE OF A (KSI)
100 REM C9=AVG VALUE OF C (KSI)
110 REM KØ=FRACTURE TOUGHNESS(KSI SQRT IN)
120 REM Y=GEOMETRICAL PARAMETER
130 REM G3=MAXIMUM VALUE OF G2
140 T1=1ØØØ
150 T2=1
160 TØ=4
170 EØ=.333
180 A1=.1
190 G1=[illegible]
200 C4=1.215
210 A9=12Ø
220 C9=8Ø
230 KØ=2Ø
240 Y=1.77
250 D=[illegible]
260 LØ=Ø
270 C5=(2*A9/C9)*C4
280 P1=(TØ-2)/2
290 P2=2*EØ*P1
300 P3=1/(EØ*TØ)
310 P4=1/(2*EØ)
320 G3=(KØ**2/(A1*(C5*Y)**2))**P4
330 PRINT "G2 MAX=";G3;
340 PRINT
350 DIM F(2Ø1Ø),G(2ØØØ)
```

ACCELERATED RANDOM TEST
LEVEL COMPUTATION PROGRAM
(BASIC LANGUAGE)
CONTINUED

```
360 V5=G1*(T1/T2)**P3
370 H5=((A1*(C5*Y)**2)/KØ**2)**P1
380 L5=1-H5*G1**P2
390 FOR J=1 TO D
400 P(J)=H3*(J/(D+1))
410 B5=((1/L5)-((H5/L5)*P(J)**P2))
420 Q(J)=P(J)-V5*(B5**P3)
430 NEXT J
440 FOR J=2 TO D
450 R5=Q(J)/Q(J-1)
460 IF R5>Ø GO TO 48Ø
470 GO TO 52Ø
480 NEXT J
490 PRINT "NO SIGN CHANGE;NO SOLUTION"
500 LØ=1
510 GO TO 55Ø
520 M1=Q(J)*P(J-1)+P(J)*ABS(Q(J-1))
530 M2=Q(J)+ABS(Q(J-1))
540 G2=M1/M2
550 PRINT "T1,T2,THETA,Y"
560 PRINT T1,T2,TØ,Y
570 PRINT "A1,DELTA K,G1"
580 PRINT A1,KØ,G1
590 PRINT "ETA,C4,A BAR,C BAR"
600 PRINT EØ,C4,A9,C9
610 PRINT
620 IF LØ=1 GO TO 71Ø
630 R1=T1/T2
640 R2=G2/G1
650 XØ=R1/(R2*TØ)
660 PRINT "TEST TIME COMPRESSION FACTOR=";R1
670 PRINT "CORRECTION FACTOR,X=";XØ
680 PRINT "ACCEL LEVEL RATIO=";R2
690 PRINT
700 PRINT "TEST ACCEL LEVEL (G RMS)=";G2
710 END
```

LOW CYCLE FATIGUE

The previous development of fracture mechanics effects was restricted to elastic stress fields where the use of the stress intensity factor ΔK and fracture toughness ΔK_c are well established and readily applied. The crack growth rate expression used was that of Paris [7]:

$$\frac{da}{dN} = c_o \, \Delta K^{\theta} \tag{31}$$

where $$\Delta K = Y \, \Delta S a^{1/2} \tag{32}$$

The high strain fatigue (i.e. low cycle fatigue region) crack growth rate characterization is not as well established.

The most accurate characterization is the J-Integral explored by Dowling and Begley [8] [9] [10]. J is a line integral.

The Dowling and Begley expression is:

$$\frac{da}{dN} = C_1 \, \Delta J^{\gamma} \tag{35}$$

where ΔJ is the range of the energy line integral J, and C_1 and γ are material dependent constants. This expression has general use to all materials [12].

Mowbray [10] has shown that this relationship also reduces to the Coffin-Manson low cycle fatigue expression. An important aspect of the Dowling and Begley work is that only the loading during crack face opening results in damage.

At present there is one objection to applying the J-Integral to fatigue crack growth and that pertains to the mathematical definition of J [11]. It is mathematically valid within the limits of deformation plasticity theory, which precludes unloading. Dowling approaches this objection on the basis that J may have more applicability than the current mathematical definition indicates. More test data will help resolve this issue.

There is difficulty at present with the practical application of the approach, the determinition of J versus crack length "a" relationships [11]. There are only a limited number of configurations for which J is known or can be directly measured. However, any approach involving non-linear material behavior will have similar difficulties.

The previously developed elastic accelerated test level equations are in terms of the Paris equation parameters; namely, c_o , ΔK and θ . Those same equations can be used to determine the inelastic accelerated test levels by substituting as follows:

TABLE VI EQUIVALENT PARAMETERS

IN-PLACE OF	SUBSTITUTE DOWLING-BEGLEY
ΔK	ΔJ
ΔK_c	ΔJ_c
c_o	C_1
θ	γ

SINE-RANDOM EQUIVALENCY

Appendix E (SINE-RANDOM EQUIVALENCE DERIVATION) derives the relationship between the sinusoidal "black box" vibe input level $\ddot{x}_S$ and the random vibe input power spectral density W_o in the vicinity of structural resonance that will cumulate the same fatigue damage in the same test time. The desired expression is

$$\ddot{x}_S^{\,n_S} = (1.25)\left(\frac{\bar{A}}{\bar{C}}\right)\sqrt{\frac{f_n}{Q}}\left[\frac{1}{f_b - f_a}\right]^{\frac{1-n_R}{2}} W_o^{\,n_R/2} \qquad (36)$$

In general there is no single, unique relationship between $\ddot{x}_S$ and W_o . From a fracture mechanic's viewpoint *initial flaws of length* a_i *do not* alter the above equivalence expression. A typical example is worked out in Appendix E .

MULTI-DEGREE-OF-FREEDOM SYSTEMS

All of the previously developed equations have expressed damage state and rate parameters in terms of a stress and stress cycles per time. These same equations can also be used for multi-degree-of-freedom (MDF) systems by using the proper damage state and rate parameters. Reference [13] shows that the proper damage state and rate parameters are obtained by adding the various resonant mode stresses and resonant frequencies in the mean-square sense.

Consider the example of a two-degree-of-freedom (2DF) system whose stress response is shown in figure 8 .

f_1 = center frequency of first resonant mode stress response (Hz)

f_2 = center frequency of second resonant mode stress response (Hz)

$S(f)$ = stress power spectral denisty (KSI^2/Hz)

$\sigma_1{}^2$ = mean-square stress response of the first resonant mode (KSI^2)

$\sigma_2{}^2$ = mean-square stress response of the second resonant mode (KSI^2)

σ_T = effective damage state stress (KSI)

f_{eff} = effective damage rate (Hz)

$$\sigma_1{}^2 = \int_{f_a}^{f_b} S_1\,(f)\,d\,f \qquad (37)$$

$$\sigma_2{}^2 = \int_{f_c}^{f_d} S_2\,(f)\,d\,f \qquad (38)$$

$$\sigma_T = \sqrt{\sigma_1^{\,2} + \sigma_2^{\,2}} \qquad (39)$$

$$f_{eff} = \sqrt{\frac{\sigma_1^{\,2}}{\sigma_T^{\,2}}\, f_1^{\,2} + \frac{\sigma_2^{\,2}}{\sigma_T^{\,2}}\, f_2^{\,2}} \qquad (40)$$

$$f_1 < f_{eff} < f_2$$

f_{eff} will take on a value nearest the resonant mode having the larger stress power.

$$N_{eff} = f_{eff} \times T \qquad (41)$$

where T = test time

N_{eff} = number of effective stress cycles

The above 2DF case can be extended to the MDF case as follows:

$$\sigma_T = \sqrt{\sum_{j=1}^{k} \sigma_j^{\,2}} \qquad (42)$$

$$j = 1, 2, 3 \ldots k$$

where j = resonant mode index

k = total number of resonant modes

$$f_{eff} = \sqrt{\sum_{j=1}^{k} \left(\frac{\sigma_j}{\sigma_T}\; f_j\right)^2} \qquad (43)$$

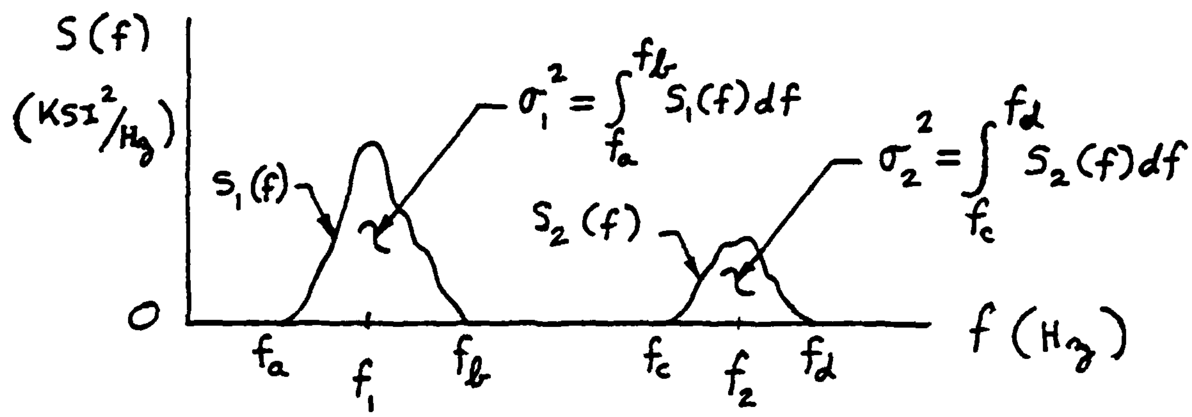

FIGURE 8 2DF STRESS POWER SPECTRAL DENSITY PLOTS

RANDOM VARIABLE CONSIDERATIONS

The results of this study can be characterized as being deterministic. That is, in all cases the applied stresses and fatigue curves were treated as deterministic, not as random variables. It is beyond the scope of this study to go into the details of how to treat the random variable case. It will be pointed out, however, that methods have been developed (e.g. references [1], [2], [7], [13]) for the random variable case. The values of input acceleration levels and test time must be converted to stresses and stress cycles respectively. The stress and cycles should then be treated as median or average values. Standard deviation stress and fatigue curve values must then be assigned. Then the random variable expressions (e.g. probability of failure versus cycles) can be used.

EXAMPLE:

Given: 63 - 37 Tin-Lead Solder

$\eta = 1$

$C_5 = 0.292$ KSI/G (SHEAR STRESS)

$\ddot{x}_2 = 7$ g rms at accelerated test level

$W_o = 0.025\ g^2/Hz$

Quantity of solder joints being stressed = 100

$f_n = 200$ Hz

$T_2 = 25$ minutes total

Find: Average number of cumulative solder joint failures versus test time.

Solution: $\tau = 0.292\ \ddot{x}_2 = 2.04$ KSI RMS shear

$N = 12 \times 10^3\ T$ cycles

$\bar{C} = 6.62$ KSI (See page E-12)

$\beta = 8.97$

$$N_m = \left(\frac{\bar{C}}{\tau}\right)^{\beta} = 3.6 \times 10^4 \text{ cycles}$$

$T_m = 3$ minutes

Choose the fatigue curve standard deviation to be 10% of the median value (i.e. $\bar{A}/\Delta = 10$)

$$F(T) = 0.5 + \operatorname{erf}\left[\frac{\bar{A}}{\Delta}\left\{\left(\frac{T}{T_m}\right)^{1/\beta} - 1\right\}\right] \quad (44)$$

$$F(T) = 0.5 + \operatorname{erf}\left[10\left\{\left(\frac{T}{3}\right)^{0.112} - 1\right\}\right] \quad (45)$$

$$\bar{q}(T) = 100 \times F(T)$$

where

Δ = standard deviation of fatigue curve average value $\bar{A}$.

T = test time (minutes)

T_m = test time for 50% (median) solder joint failures

$\bar{q}(T)$ = average number of cumulative solder joint failures as a function of time.

$$\operatorname{erf}(\alpha) = \frac{1}{\sqrt{2\pi}}\int_0^{\alpha} e^{-y^2/2}\,dy \quad (46)$$

Figure 9 is a plot of $\overline{q}(T)$ versus T . About 92% of the solder joint failures will occur after 10 minutes of tests.

If this analysis were deterministic (i.e. $\Delta = 0$), all failures would have occured at $T_2 = T_m = 3$ minutes. The scatterband of fatigue curve failure points results in failures occuring both before and after T_m .

Given: Same example as the previous one involving 63-37 Tin-Lead Solder.

Find: Equivalent resonance dwell sinusoidal input acceleration level $\ddot{x}_s$ that will produce the same quantity of failures versus time as shown in figure 9.

Solution: From data on page E-12

$$\frac{\overline{A}}{\overline{C}} = 2.19$$

Using equation (98)

$$\ddot{x}_s = 6.12/Q^{\frac{1}{2}} \quad g's$$

Q	$\ddot{x}_s$ (g's)
10	1.94
20	1.37
30	1.12
20	0.97
50	0.87

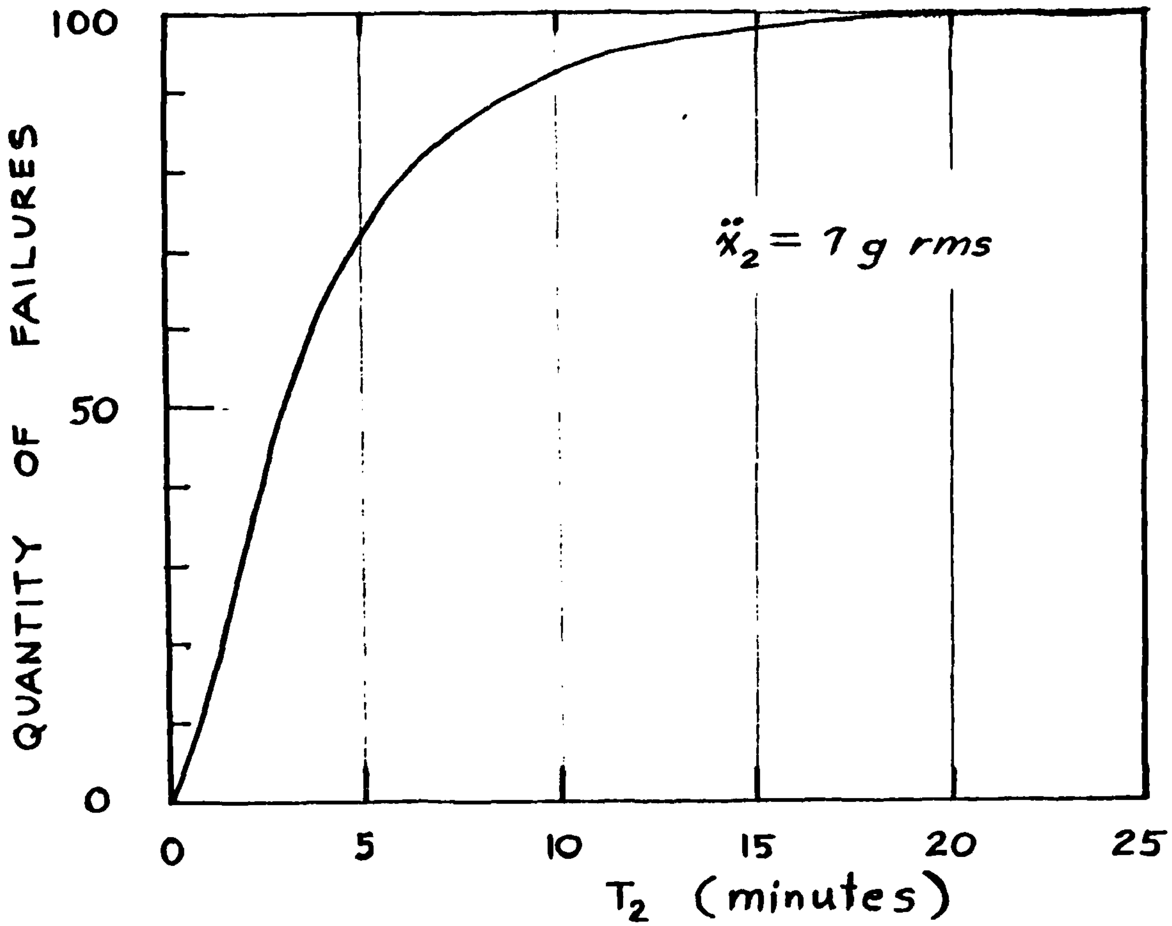

FIGURE 9 FAILURE DISTRIBUTION FOR SOLDER JOINT EXAMPLE

MULTI ACCELERATION FACTOR CONSIDERATIONS

In a relatively complex structural assembly such as a typical electronic "black box" more than one acceleration factor can be computed; one for each structural element that could experience a fatigue failure. Even when only the more highly stressed elements are given consideration, the problem still exists of choosing the single acceleration factor to be used for testing the "black box."

At the service environment the various structural elements that could fail will each possess its individual cumulative damage value (i.e. potential for failure). Due to these cumulative damage values in conjunction with scatterband effects one structural element will fail before the others.

If all structural elements had the same acceleration factor, the cumulative damage and scatterband relationship between elements would remain unaltered at the accelerated test environment. This would be the case whether the acceleration factor included non-linear parameters in its estimation or not. The same element would fail before the others at both the accelerated and service environments. Thus, scatterband parameters are not important in estimating the acceleration factor. Only the median parameter values influence the acceleration factor value.

In general all the structural elements will not have the same acceleration factor. Differences in damping linearity, fatigue and crack growth rate curve slopes, and initial flaw sizes (η, β, θ, a_i) as examples will alter the cumulative damage relationship between the structural elements. Which of the various acceleration factors is chosen will result in a proper accelerated test for only one class of structural elements. The other elements will either be under or over-tested. The most conservative approach would be to select the largest acceleration factor for testing the black box. The acceleration factor used could be the average of all acceleration factors. Thus the selection of a single acceleration factor is considered to be subjective.

DECELERATION FACTOR CONSIDERATIONS

It is sometimes of interest to decelerate a test instead of accelerating one. An example would involve a "black box" that had been vibration qualified at a relatively large input level $\ddot{x}_1$ for a short duration T_1. It might be desired to compute an acceptance test which would produce the same damage (i.e. potential for failure) but at a lower level $\ddot{x}_2$ for a longer duration T_2. In this case $\ddot{x}_1 > \ddot{x}_2$ and $T_2 > T_1$. The desired computation can proceed using previously developed equations.

For the fatigue case (i.e. the initial flaw size $a_i = 0$) equations (17) or (111) can be used as follows depending upon whether $\ddot{x}_2$ or T_2 is the unknown quantity:

$$T_2 = T_1 \left(\frac{\ddot{x}_1}{\ddot{x}_2} \right)^{1/\eta\beta}$$

$$\ddot{x}_2 = \ddot{x}_1 \left(\frac{T_1}{T_2} \right)^{1/\eta\beta}$$

For the fracture mechanics case (i.e. $a_i > 0$) equations (105) and (106) and program PL-3 are applicable. Equations (105) and (106) should be used if T_2 is the unknown quantity. PL-3 should be used if $\ddot{x}_2$ is the unknown quantity. Care should be taken that $\ddot{x}_1 > \ddot{x}_2$ and $T_2 > T_1$. T_1 and T_2 can be in any time units (e.g. seconds, minutes) so long as T_1 has the same units as T_2. As mentioned in a previous section if the value of a_i is chosen too large, X will be negative and PL-3 won't execute. A negative value of X means that $a_i > a_{c_1}$ or a_{c_2}. That is, the part will fracture immediately.

PL-3 deceleration factor example:

Given: 7075-T6 alloy

a_i = 0.01 inches

$\ddot{x}_1$ = 15 G rms (Qual Test Level)

T_1 = 1 minute

T_2 = 100 minutes

Find: Acceptance Test Level $\ddot{x}_2$ for η = 0.833, 1

Using PL-3

η	$\ddot{x}_2$ (G rms)
0.833	4.02
1	5.61

As in previous examples the deceleration factor is sensitive to the damping linearity.

For a complex "black box" where multi-deceleration factors can exist the choice of a single factor is subjective. The most conservative approach would be to select the smallest deceleration factor.

CONCLUDING REMARKS

1. Expressions have been developed that relate service environmental level and duration to accelerated test level and compressed duration for the same potential to do fatigue damage. The power law expression commonly used in the industry is shown to apply on for those fatigue cases where initial flaws (i.e. cracks) of length a_i in the structural elements being stressed do not exist. Fracture Mechanics effects (i.e. where cracks, either actual or hypothesized, already exist) complicate the expression; it becomes a transcendental function whose solution is most easily handled by the included Basic Language computer program. Levels are in terms of either stress or "black box" vibe input acceleration. Durations are in terms of either number of applied stress cycles or time.

2. The developed expressions apply to sine or random vibration and thermal cycling for both the low and high cycle (i.e. inelastic and elastic) fatigue regions. Linear and non-linear dependence of stress upon input vibration level is included. Single and multi-degree-of-freedom systems are also included.

3. The developed expressions are summarized in Appendix G.

4. All equations are in practical engineering terms and are expected to be accurate. Application is straightforward.

5. Random variables (e.g. scatterband fatigue curve and applied stress) can be added to the results of this basically deterministic study.

6. The damage state and rate (i.e. conditions of similitude) must be the same for both service and accelerated test environments.

7. Fatigue damage is directly related to the stress level and number of applied stress cycles in a structural element. It is only indirectly related to the "black box" vibe input acceleration level and test duration. The random response stress is directly related to the value of the vibe input acceleration power spectral density W_o in the vicinity of the resonant frequency. It is only indirectly related to the overall "black box" vibe input rms acceleration level $\ddot{x}$.

8. Two examples of quantitatively compensating for similitude condition violations are given.

9. The power law relation applies only for $a_i = 0$. The value of the power law exponent is $1/\eta\beta$. $\beta \approx 2$ in the low cycle fatigue region for most structural materials. In the high cycle fatigue region $\beta \approx 9$ for ductile materials and ≈ 20 for brittle materials. η represents the damping linearity. $\eta = 1$ (linear). $0.714 \leq \eta \leq 2$ for the cases studied.

10. An initial flaw reduces fatigue life. Specifically it alters the form of the fatigue curve. A typical fatigue curve is of the form:

$$\frac{\Delta S}{2} = \bar{A} \; N^{-1/\beta} \qquad \sigma_i = \bar{C}_i \; N^{-1/\beta} \qquad (47)$$

The modified form is

$$\bar{C}_i = \frac{\bar{C}}{\bar{A}} \; \bar{A}_i$$

$$\frac{\Delta S}{2} = \bar{A}_i \; N^{-1/\theta} \qquad (48)$$

$$\bar{A}_i < \bar{A} \quad ; \quad \theta < \beta$$

$$\bar{A}_i = \frac{\text{constant}}{a_i^{\;0.5 - 1/\theta}}$$

For one example using 7075-T6 with $a_i = 0.007$ inches

$$\bar{A} = 180 \text{ KSI} \quad ; \quad \beta = 9.65$$

$$\bar{A}_i = 111 \text{ KSI} \quad ; \quad \theta = 4$$

11. An initial flaw does not alter the relationship between the sine and random stress levels that will propagate a crack of the same size in the same time; namely,

$$\Delta S = \left(\frac{2\bar{A}}{\bar{C}}\right)\sigma \qquad (a_i = 0) \qquad (49)$$

$$\frac{\bar{A}_i}{\bar{C}_i} = \frac{\bar{A}}{\bar{C}} \qquad (50)$$

12. The random vibe transcendental function is

$$\ddot{x}_2 - \ddot{x}_1\left(\frac{T_1}{T_2}\right)^{\frac{1}{\eta\theta}} \left(\frac{1}{X}\right)^{\frac{1}{\eta\theta}} = 0 \qquad (51)$$

where X is a function of both $\ddot{x}_2$ and $\ddot{x}_1$

This equation cannot be normalized in closed form fashion as in terms of $(\ddot{x}_2/\ddot{x}_1)$ and (T_1/T_2) due to the inherent non-linearities in the Fracture Mechanics terms.

13. A J-Integral fracture mechanics equation is proposed for use in the low cycle fatigue region. It has an identical crack growth rate equation form to the one used in the high cycle fatigue region. Thus, its parameter values can be substituted into the previously developed high cycle equations.

14. In general there is no single, unique relation between the sine input vibe acceleration level $\ddot{x}_S$ and the random vibe input power spectral density W_o. The relationship can be altered as a function material ductility, damping linearity, and f_n/Q ratio.

15. The developed equations also apply to multi-degree-of-freedom systems by using the proper damage state and rate parameters. These parameters are obtained by adding the various mode resonant response stresses and resonant frequencies in the mean-square sense. For a 2DF system:

$$\sigma_T = \sqrt{\sigma_1^2 + \sigma_2^2}$$

$$f_{eff} = \sqrt{\frac{\sigma_1^2}{\sigma_T^2} f_1^2 + \frac{\sigma_2^2}{\sigma_T^2} f_2^2}$$

$$f_1 < f_{eff} < f_2$$

$$f_{eff} \approx f_1 \text{ if } \sigma_1 >> \sigma_2$$

$$f_{eff} \approx f_2 \text{ if } \sigma_2 >> \sigma_1$$

16. The selection of a single acceleration or deceleration factor for a complex "black box" is considered to be subjective in general. The most conservative approach would be to select the largest acceleration factor and the smallest deceleration factor.

SYMBOLS

a	crack half-length
a_c	critical value of a
a_i	initial value of a
A	cross-sectional area
$\overline{A}$	material sine fatigue curve constant
$\overline{A}_i$	material sine fatigue curve constant with $a_i > 0$
B	random stress bandwidth
c	viscous damping coefficient
c_o	constant of Paris crack growth rate curve
$\overline{C}$	material random fatigue curve constant
$\overline{C}_i$	material random fatigue curve constant with $a_i > 0$
C_1	constant of Dowling-Begley crack growth rate curve
$C_1 - C_5$	constants
D_j	cumulated damage at the j^{th} stress level
2DF	two-degree-of-freedom
$\frac{da}{dN}$	crack growth rate
E	modules of elasticity
erf	Error Function
f	frequency
f_{eff}	effective frequency
f_n	resonant frequency
F_f	Coulomb friction force value
$F(T)$	cumulative probability of failing in time T
g's	gravity acceleration units

h	thickness
H	normalized stress limit level
I	area moment of inertia
j	index
J_o	constant of specific damping energy curve
J	energy line integral
ΔJ	range of J
ΔJ_c	critical value of ΔJ
k	total number of resonant modes, spring stiffness
ksi	thousands of pounds per square inch
K_v	volume stress factor
ΔK	stress intensity factor range
ΔK_c	fracture toughness
ℓ	length
L	stress limit level
m	mass
n	exponent of specific damping energy curve
N	number of stress cycles
N_f	number of stress cycles to failure
N_{opp}	number of independent opportunities for stress peak occurrence
p	probability
P	load
PSD	power spectral density
Q	resonant amplification factor
$\overline{q}(T)$	average number of cumulative failures
S	sinusoidal stress amplitude
ΔS	sinusoidal stress range

$S(f)$	power spectral density as a function of frequency
t_i	average time between independent events
T	time
$\overline{T}$	average time between stress peaks
w	width
W	weight
W_o	acceleration power spectral density
$\ddot{x}$	acceleration level
y	dummy variable
Y	geometrical parameter
$\ddot{y}$	response acceleration
z	relative displacement
rms	root-mean-square
X	fracture mechanics dependent term
α	dummy variable
β	fatigue curve slope parameter
Δ	standard deviation of $\overline{A}$
σ	random rms stress value
θ	constant of Paris crack growth rate curve
$\Gamma(\alpha)$	Gamma Function with argument α
$\gamma(\alpha)$	Incomplete Gamma Function with argument α
ε	applied strain amplitude
ε'_f	fatigue ductility coefficient
ε_u	material ultimate percent elongation
$\Delta\varepsilon$	applied strain range
$\Delta\varepsilon_p$	average net section plastic strain range
$\Delta\varepsilon_e$	average net section elastic strain range

η damping linearity term

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APPENDIX A

SIMILITUDE COMPENSATION EXAMPLES

The equations developed in this study to calculate test levels to cumulate the same fatigue damage per the desired time compression ratio are accurate only if the conditions of similitude (i.e. damage state and rate) are the same in both environments. There are many common ways for such conditions to be violated. Only one such violated condition and a technique to compensate for the violation will be treated in this section.

Reference [1] shows that most fatigue damage during random vibration is caused by stress peaks between 2σ to 5σ (where σ = rms stress level) for ductile materials and between 3σ to 6σ for brittle materials. If the peak stresses are limited from exceeding some limit level L (KSI), the fatigue life will be extended from that for no stress limiting (i.e. $L = \infty$).

Electronic equipment mounted in rocket propelled spacecraft or jet aircraft experience "unlimited" stress peaks. Note: for a frequency band-limited process the very high (i.e. $> 6\sigma$) peaks do not occur very often even when limiting is not present. For a typical stress response process that is band-limited to 500 Hz: 6σ peaks occur about every 36 hours and 7σ peaks occur about every 3 years on the average. Electrodynamic shaker systems also produce high peaks in a gaussian fashion due to peak restoration by the shaker transfer function as long as the noise generator voltage is hard limited (i.e. clipped) no lower than 3σ. Such peaks may not occur in a test of relatively short duration. See Appendix H.

Stress limiting will occur if the noise generator voltage of an electrodynamic shaker system is clipped below 3σ , if the shaker system (mechanical or otherwise) is incapable of producing 5 or 6σ peaks conceivably due to Brinelling of metal surfaces or compressibility of fluids, or if the structural element's motion being vibrated is snubbed by design.

The effective limit level is L/σ .

Define $$H = \frac{1}{2}\left(\frac{L}{\sigma}\right)^2 \quad ; \quad \alpha = \frac{2+\beta}{2} \qquad (52)$$

where L = limit stress level (KSI)

σ = rms stress level (KSI RMS)

β = material fatigue curve slope parameter

N_f = fatigue life (i.e. cycles to failure) for no limiting (i.e. $L = \infty$)

N_{f_L} = extended fatigue life with stress limiting

From reference [1]

$$\frac{N_{f_L}}{N_f} = \frac{\Gamma(\alpha)}{\gamma(\alpha, H) + H^{\beta/2} e^{-H}} \qquad (53)$$

$\Gamma(\alpha)$ = Gamma Function

$\gamma(\alpha)$ = Incomplete Gamma Function

$$= \int_o^H z^{\alpha-1} e^{-z}\, dz \qquad (54)$$

The following table shows the factor of fatigue life extension for various limit level on Copper wire.

TABLE VII LIMIT LEVEL FATIGUE LIFE EXTENSION

Limit Level (L/σ)	N_{f_L} / N_f
∞	1
4	1.08
3	1.86
2	13.0

For the purposes of this analysis assume that no limiting occurs at service vibration levels but that $L/\sigma = 3$ at the accelerated test level. The compensation technique is to extend the desired test time by the factor N_{f_L}/N_f. In the above example T_2 would be multiplied by 1.86.

A second type of similitude condition violation compensation method will be treated.

Given: The similitude violation is due to a difference in stress spectra between service and test environments. The stress vibration system is 2DF. The calculated acceleration factor $(\ddot{x}_2/\ddot{x}_1)$ is 3 for the desired time compression factor (T_2/T_1) using equation (111). The spectra parameter values are given below and in Figures 10 and 11:

PARAMETER	ENVIRONMENT	
	SERVICE	TEST
σ_a (ksi)	8	24
σ_b (ksi)	16	20
f_a (Hz)	150	150
f_b (Hz)	375	375
σ_T (ksi)	17.9	31.2
f_{eff} (Hz)	342	264

Find: The appropriate test compensation factors such that $\sigma_{T_{TEST}} = 3\,\sigma_{T_{SERVICE}}$ and $f_{eff_{TEST}} = f_{eff_{SERVICE}}$.

Solution: It can be seen that the resonant frequencies are the same at both environments. However, σ_b did not increase from 16 to 48 ksi as desired. This would cause an inappropriate test damage state and rate. $\sigma_{T_{TEST}}$ should be 3 x 17.9 = 53.7 ksi. Therefore, $\ddot{x}_2$

needs to be increased by an additional factor of 53.7/31.2 = 1.72. The test duration needs to be increased from its computed compressed value by a factor of 342/264 = 1.3.

Alternative Solution:

In some cases it is possible to increase σ_b at the test environment from 20 ksi to the desired value of 48 ksi by the use of shaker equalization techniques. The spectra obtained is the desired test spectra. No further compensation to overall rms level or test duration is required. This is the most direct and preferred solution.

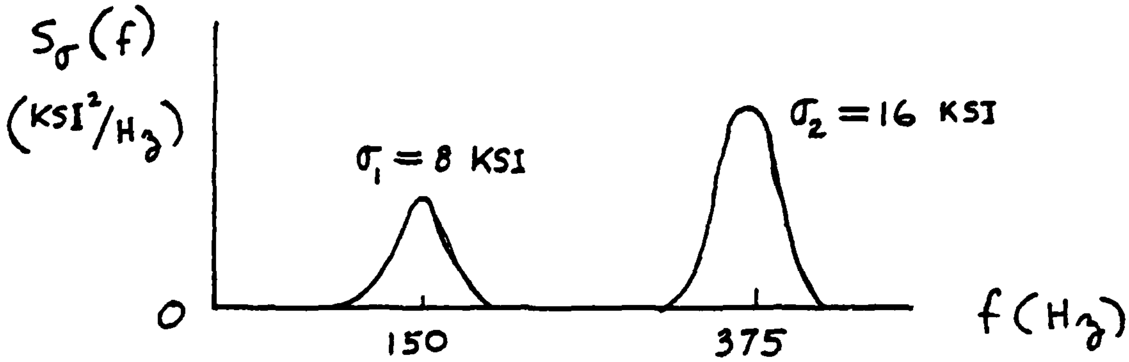


FIGURE 10 SERVICE LEVEL STRESS SPECTRUM

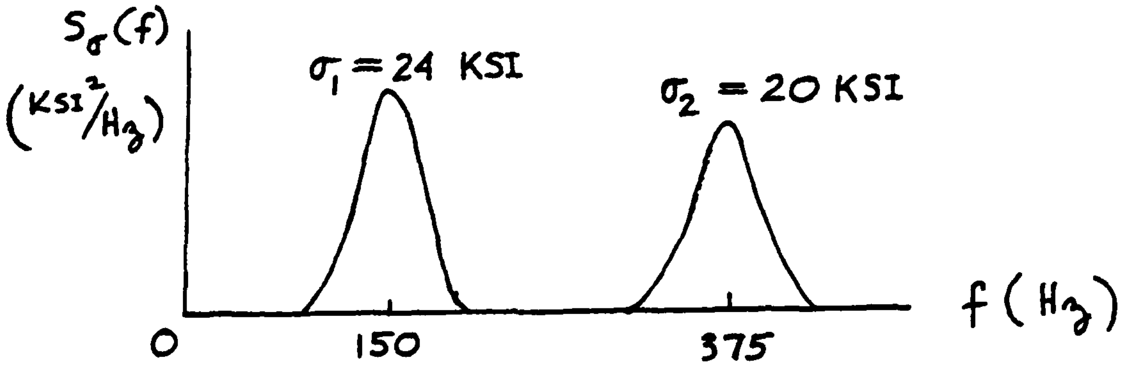


FIGURE 11 TEST LEVEL STRESS SPECTRUM

APPENDIX B

DAMPING TERM DERIVATION

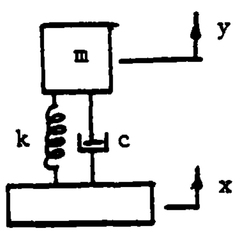
The power law exponent is called the damping term η when related the stress at an internal structural element due to the "black box" input vibe acceleration level:

$$\text{SINE:} \quad \frac{\Delta S}{2} = S = C_2 \, \ddot{x}_S^{\;\eta_S} \tag{55}$$

$$\text{RANDOM:} \quad \sigma = C4 \, \ddot{x}_R^{\;\eta_R} \tag{56}$$

The relationship between stress and acceleration is linear for $\eta = 1$. Non-linearities in this relationship can arise if any of the elements of the idealized mass-spring-dashpot system becomes amplitude sensitive (i.e. non-linear). Several such cases will be evaluated. The technique used in reference [14] for developing the η determination will be used here.

CASE A (Q = constant: sinusoidal)



$z = y - x$

S = stress

A = spring cross-sectional area

$$\frac{\ddot{y}}{\ddot{x}} = \frac{y}{x} = Q \tag{57}$$

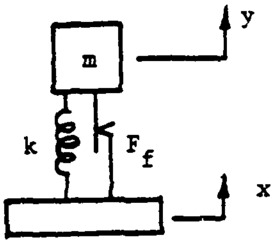
At resonance:

$$m\ddot{y} = SA \; ; \; mQ\ddot{x} = SA$$

$$S = \frac{mQ}{A} \ddot{x}$$

$$\eta_S = 1$$

CASE B (Coulomb Friction: sinusoidal)



For $Q > 3$; Reference [15]

$$Q = \frac{2k\ddot{x}}{F_f \, 4\pi^2 \, f_n^2} \tag{58}$$

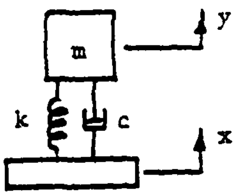
$$m\ddot{y} = mQ\ddot{x} = SA$$

$$\ddot{x} = \frac{SA}{mQ} \quad \frac{SAF_f \, 4\pi^2 \, f_n^2}{2 \, m \, k \, \ddot{x}} \quad ; \quad S = \frac{2 \, k \, m}{A \, F_f \, 4\pi^2 \, f_n^2} \, \ddot{x}^2 \tag{59}$$

$$n_S = 2$$

CASE C (internal sinusoidal stress-strain hysteresis)

From reference [14], [16]



$$z = y - x$$

$$S = C_1 \, z$$

$$Q = \frac{Kv \, \pi \, S^2}{E \, J_o S^n} \quad ; \quad n = 2.4 \text{ for most structural materials stressed below 0.8 of fatigue strength} \tag{60}$$

$$m\ddot{y} = mQ\ddot{x} = SA \; ; \quad n = 8 \text{ for higher stresses}$$

$$\ddot{x} = \frac{SA}{mQ} = \frac{A \, E \, J_o S^{n+1}}{m \, Kv \, \pi \, S^2} = \frac{A \, E \, J_o S^{n-1}}{m \, Kv \, \pi} \tag{61}$$

$$S = \left[\frac{m \, Kv \, \pi}{A \, E \, J_o}\right]^{\frac{1}{n-1}} \quad \ddot{x}^{\frac{1}{n-1}} \tag{62}$$

$$S = C_1 \, \ddot{x}^{0.714} \quad \text{for } n = 2.4 \tag{63}$$

$$n_S = 0.714$$

CASE D (internal random stress-strain hysteresis)

Same figure as for CASE C

$$\ddot{z}_{rms} = \ddot{y}_{rms}$$

$$m\ddot{y}_{rms} = \sigma A$$

$$\ddot{y}_{rms} = \sqrt{\frac{\pi}{2} f_n W_o Q} \quad ; \quad \ddot{x}_{rms} = \sqrt{W_o (f_b - f_a)}$$

$$\frac{\ddot{y}_{rms}}{\ddot{x}_{rms}} = \sqrt{\frac{\pi}{2} \frac{f_n Q}{(f_b - f_a)}}$$

$$\ddot{x}_{rms} = \frac{A\sigma}{m\sqrt{\frac{\pi}{2} \frac{f_n}{(f_b-f_a)}}} \left[\frac{E J_o \sigma^n}{Kv \pi \sigma^2}\right]^{1/2}$$

$$\ddot{x} = \left\{ \frac{A (EJ/Kv \pi)^{1/2}}{m \frac{\pi}{2} \left[\frac{f_n}{(f_b-f_a)}\right]^{1/2}} \right\} \sigma^{n/2}$$

$$\sigma = \frac{m}{A} \left[\frac{\pi^2 f_n Kv}{2EJ_o (f_b-f_a)}\right]^{1/n} \ddot{x}^{2/n} \qquad (64)$$

$$\eta_R = \frac{2}{n} = \frac{2}{2.4} = 0.833$$

$\eta_R = 0.833$ ⇦ for structural materials

For viscoelastic adhesives stressed in shear [14]

$\eta_R = \frac{2}{2.55} = 0.784$ ⇦

Figure 12 shows the non-linear relationship between σ and $\ddot{x}$ for C4 = 1.0 KSI/G .

CASE E (Coulomb Friction: random)

Same figure as for CASE B

$$\ddot{z}_{rms} \approx \ddot{y}_{rms}$$

$$m\ddot{y}_{rms} = \sigma A$$

$$\ddot{y}_{rms} = \sqrt{\frac{\pi}{2} f_n W_o Q} \quad ; \quad \ddot{x}_{rms} = \sqrt{W_o (f_b - f_a)}$$

$$\ddot{y}_{rms} = \sqrt{\frac{\pi}{2} \frac{f_n Q}{(f_b - f_a)}} \qquad \ddot{x}_{rms} = \frac{\sigma A}{m}$$

$$Q \approx \frac{2 k \ddot{x}}{F_f 4\pi^2 f_n^2}$$

$$\therefore \quad \sigma = \left[\frac{m}{A} \sqrt{\frac{k}{F_f (f_b - f_a) 4\pi f_n}} \right] \ddot{x}^{1.5} \tag{65}$$

$$\eta_R = 1.5$$

CASE F (Viscoelastic Materials: sinusoidal)

Same figure as for CASE C

For viscoeleastic materials stressed in shear [14] n = 2.55. From equation (62)

$$\eta_S = \frac{1}{n - 1} = \frac{1}{2.55 - 1}$$

$$\eta_S = 0.645$$

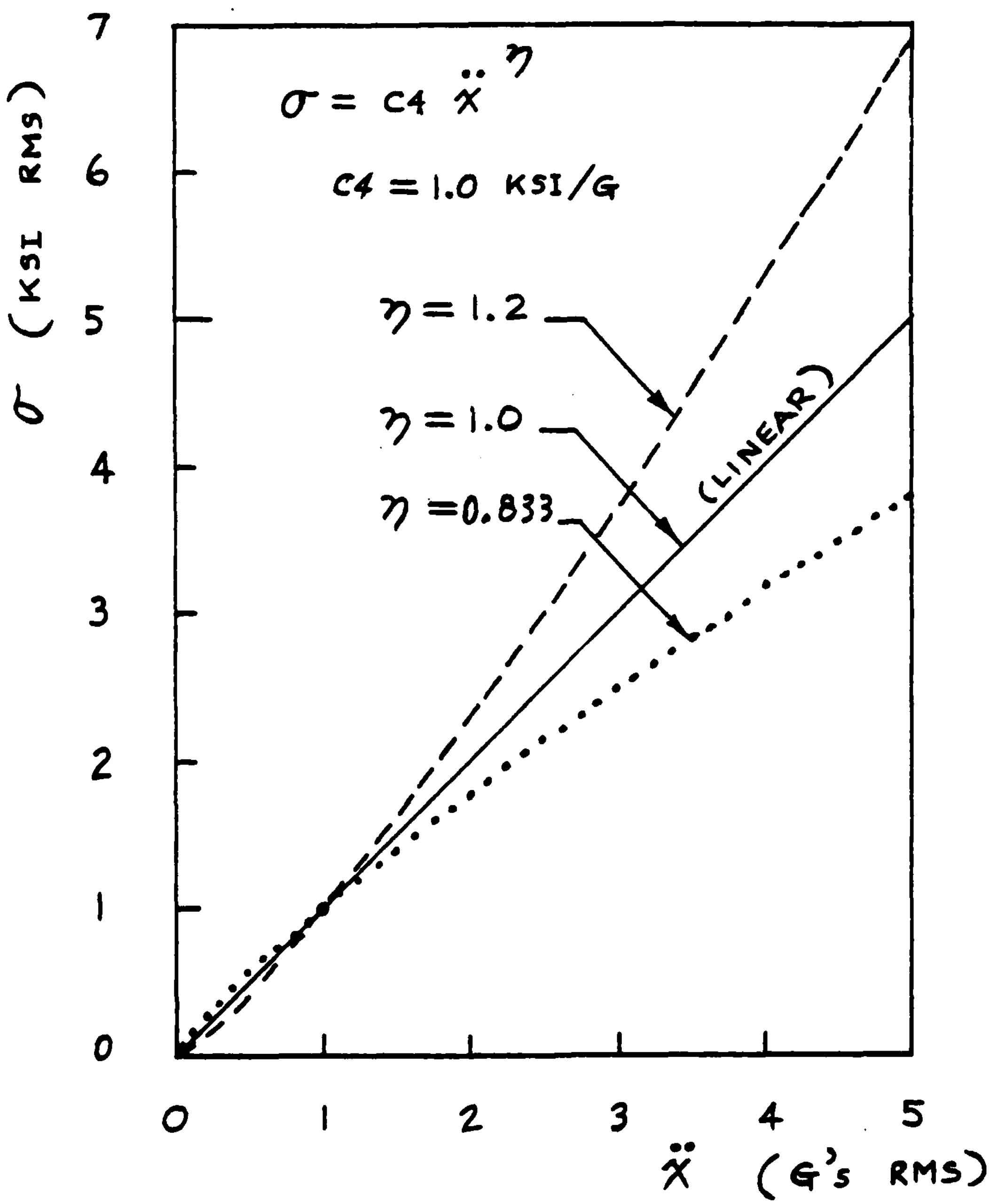


FIGURE 12 NON-LINEAR $\sigma - \ddot{x}$ RELATIONSHIP

APPENDIX C

FRACTURE MECHANICS FATIGUE CURVE PARAMETER APPROXIMATIONS

$$N_f \approx \frac{2}{(\theta - 2)\ c_o\ Y^{\theta}\ \Delta S^{\theta}} \left(\frac{1}{a_i}\right)^{\frac{\theta - 2}{2}} \qquad \text{from reference [7]} \qquad (66)$$

$$\frac{\Delta S}{2} \approx \left[\frac{2}{(\theta - 2)\ c_o\ Y^{\theta}\ N_f\ 2^{\theta}} \left(\frac{1}{a_i}\right)^{\frac{\theta - 2}{2}}\right]^{1/\theta} \qquad \text{ksi} \qquad (67)$$

Define

$\bar{A}_i$ = sinusoidal curve "y - intercept" parameter with an initial flaw of length a_i

$$\bar{A}_i = \left.\frac{\Delta S}{2}\right|_{N_f = 1}$$

$$\bar{A}_i \approx \left[\frac{2}{(\theta - 2)\ c_o\ Y^{\theta}\ 2^{\theta}} \left(\frac{1}{a_i}\right)^{\frac{\theta - 2}{2}}\right]^{1/\theta} \qquad \text{ksi} \qquad (68)$$

$$\frac{\Delta S}{2} = \left(\frac{\bar{A}}{\bar{C}}\right)\sigma$$

$\therefore$

$$\sigma \approx \left(\frac{\bar{C}}{\bar{A}}\right)\left[\frac{2}{(\theta - 2)\ c_o\ Y^{\theta}\ N_f 2^{\theta}} \left(\frac{1}{a_i}\right)^{\frac{\theta - 2}{2}}\right]^{1/\theta} \qquad \text{ksi} \qquad (69)$$

Define

$\bar{C}_i$ = random fatigue curve "y - intercept" parameter with an initial flaw of length a_i

$$\bar{C}_i = \left(\frac{\bar{C}}{\bar{A}}\right) \bar{A}_i \quad \text{ksi} \qquad (70)$$

$$\frac{\bar{A}_i}{\bar{C}_i} = \frac{\bar{A}}{\bar{C}} \qquad (71)$$

7075-T6 $\quad 1/\theta = 0.25 \;;\; C_o = 6 \times 10^{-9}$ in/cycle ; $Y = 1.77$

$\Delta K_c = 20 \text{ KSI} \sqrt{\text{IN.}}$

$$\frac{\Delta S}{2} = \left[\frac{1.06125 \times 10^6}{a_i\, N_f}\right]^{1/\theta} \quad \text{ksi}$$

"y - intercept" $\quad \Delta S/2 = \bar{A}_i \;;\; N_f = 1$

$$\bar{A}_i = \left[\frac{1.06125 \times 10^6}{a_i}\right]^{0.25} \quad \text{ksi} \qquad (72)$$

TABLE VIII $\bar{A}_i/\bar{C}_i$ RATIO VALUES

a_i (inches)	$\bar{A}_i$ (ksi)	$\bar{C}_i$ (ksi)	$\bar{A}_i/\bar{C}_i$
.007	111	49.3	2.25
.050	67.9	30.2	2.25
.100	57.1	25.4	2.25

$\bar{A}$ = 180 ksi

$\bar{C}$ = 80 ksi

$$\frac{\bar{A}}{\bar{C}} = 2.25$$

APPENDIX D

FRACTURE MECHANICS ACCELERATED SINUSOIDAL TEST STRESS LEVEL DERIVATION

From references [1] and [7]

$$N_{f_1} = \frac{2}{(\theta - 2)\ c_o\ \Delta S_1^{\ \theta}\ Y^{\theta}} \left[\left(\frac{1}{a_i}\right)^{\frac{\theta - 2}{2}} - \left(\frac{1}{a_{c_1}}\right)^{\frac{\theta - 2}{2}} \right] \qquad (73)$$

$$N_{f_2} = \frac{2}{(\theta - 2)\ c_o\ \Delta S_2^{\ \theta}\ Y^{\theta}} \left[\left(\frac{1}{a_i}\right)^{\frac{\theta - 2}{2}} - \left(\frac{1}{a_{c_2}}\right)^{\frac{\theta - 2}{2}} \right] \qquad (74)$$

$$a_{c_1} = \left[\frac{\Delta K_c}{Y \Delta S_1}\right]^2 \quad ; \qquad a_{c_2} = \left[\frac{\Delta K_c}{Y \Delta S_2}\right]^2 \qquad (75)$$

where

N_{f_1} = number of cycles to failure at service level 1

N_{f_2} = number of cycles to failure at accelerated test level 2

θ = constant of crack growth rate curve

Y = geometrical parameter

c_o = constant of crack growth rate curve

a_i = initial crack length

a_c = critical crack length

ΔK_c = fracture toughness

ΔS = applied stress range

Define N_1 = number of applied stress cycles at level 1

N_2 = number of applied stress cycles at level 2

For equal damage $\frac{N_2}{N_{f_2}} = \frac{N_1}{N_{f_1}}$

or $\frac{N_2}{N_1} = \frac{N_{f_2}}{N_{f_1}}$

$$\frac{N_2}{N_1} = \left(\frac{\Delta S_1}{\Delta S_2}\right)^{\theta} \frac{\left[\left(\frac{1}{a_i}\right)^{\frac{\theta-2}{2}} - \left(\frac{1}{a_{c_2}}\right)^{\frac{\theta-2}{2}}\right]}{\left[\left(\frac{1}{a_i}\right)^{\frac{\theta-2}{2}} - \left(\frac{1}{a_{c_1}}\right)^{\frac{\theta-2}{2}}\right]} \tag{76}$$

$$\frac{N_2}{N_1} = \left(\frac{\Delta S_1}{\Delta S_2}\right)^{\theta} \left[\frac{1 - \left(\frac{a_i}{a_{c_2}}\right)^{\frac{\theta-2}{2}}}{1 - \left(\frac{a_i}{a_{c_1}}\right)^{\frac{\theta-2}{2}}}\right] \tag{77}$$

$$\frac{N_2}{N_1} = \left(\frac{\Delta S_1}{\Delta S_2}\right)^{\theta} \left(\frac{1}{X}\right) \tag{78}$$

where

X = correction factor; X corrects for the dependence upon a_i , Y , ΔS_1 , ΔS_2 , ΔK_c , θ

$$X = \left[\frac{1 - \left(\dfrac{a_i}{a_{c_1}}\right)^{\frac{\theta - 2}{2}}}{1 - \left(\dfrac{a_i}{a_{c_2}}\right)^{\frac{\theta - 2}{2}}}\right] \tag{79}$$

Substituting from previous equations

$$X = \left[\frac{1 - \left(\dfrac{a_i \, Y^2 \, \Delta S_1^{\,2}}{\Delta K_c^{\,2}}\right)^{\frac{\theta - 2}{2}}}{1 - \left(\dfrac{a_i \, Y^2 \, \Delta S_2^{\,2}}{\Delta K_c^{\,2}}\right)^{\frac{\theta - 2}{2}}}\right] \tag{80}$$

$$X > 1 \quad \text{for} \quad \Delta S_2 > \Delta S_1$$

$$\text{Let } h = \left(\frac{a_i \, Y^2}{\Delta K_c^{\,2}}\right)^{\frac{\theta - 2}{2}} \tag{81}$$

$$L = 1 - h \, \Delta S_1^{\,\theta - 2} \tag{82}$$

$$\frac{1}{X} = \frac{1}{L} - \frac{h}{L} \quad \Delta S_2^{\theta - 2}$$

$$\frac{\Delta S_2}{\Delta S_1} = \left(\frac{N_1}{N_2}\right)^{1/\theta} \left(\frac{1}{X}\right)^{1/\theta} \qquad (83)$$

NOTE: X is a function of both ΔS_1 and ΔS_2

$$\Delta S_2 = \Delta S_1 \left(\frac{N_1}{N_2}\right)^{1/\theta} \left(\frac{1}{X}\right)^{1/\theta} \qquad \text{Transcendental function}$$

$$\Delta S_2 - \underbrace{\Delta S_1 \left(\frac{N_1}{N_2}\right)^{1/\theta}}_{V_1} \underbrace{\left[\frac{1}{L} - \frac{h}{L} \Delta S_2^{\theta - 2}\right]^{1/\theta}}_{1/X} = 0 \qquad (84)$$

The above two equations are transcendental functions and must be solved accordingly.

$$\Delta S_2 > \Delta S_1$$

$$N_1 > N_2$$

$$X > 1$$

A negative value of X indicates that $a_i > a_{c_1}$ or $a_i > a_{c_2}$. A zero value of X indicates that $a_i = a_{c_1}$. Such situations are unrealistic for this analysis. Fracture would occur during the application of the first stress cycle of either ΔS_1 or ΔS_2.

X will be negative if the selected value for ΔS_2 is larger than

$$\Delta S_{2_{max}} = \left[\frac{\Delta K_c^{\ 2}}{a_1 \ Y^2} \right]^{1/2} \tag{85}$$

Thus the value of ΔS_2 that will satisfy the transcendental equation is

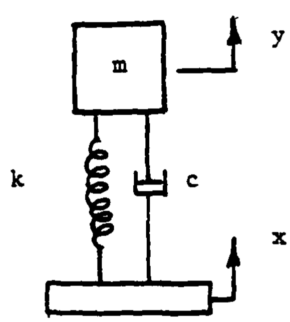
$\leq \Delta S_{2_{max}}$.

APPENDIX E

SINE-RANDOM EQUIVALENCE DERIVATION

It is sometimes desired to determine the sinusoidal input acceleration $\ddot{x}_s$ (g's) at a structural resonant frequency f_n (Hz) that will cumulate the same fatigue damage as a wide band random vibe input of power spectral density W_o (g^2/Hz). $\ddot{x}_s$ is sometimes [17] referred to as the equivalent sine input. It will also be determined if a single, unigue $\ddot{x}_s$-W_o relationship exists for all structural elements in all "black boxes". The assumptions made are that the structural elements being stressed at resonance can be characterized as single-degree-of-freedom systems and that the duration of both sine and random vibe tests are the same. Fatigue and Fracture Mechanics effects are considered.

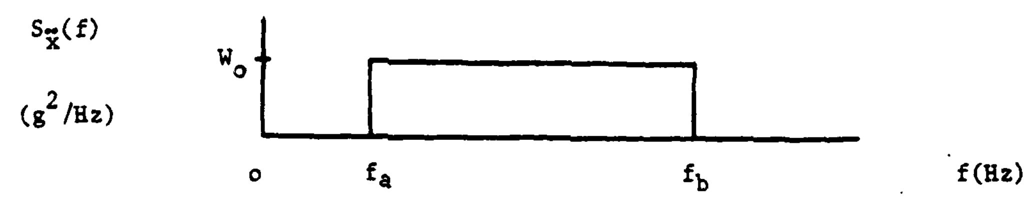
Consider the following idealized single-degree-of-freedom system:



z = strech in spring = y - x

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f_a < f_n < f_b$$



$$\ddot{x} = \int_0^{\infty} S_{\ddot{x}}(f)\,df = \sqrt{W_o(f_b - f_a)}$$

$\ddot{x}$ = random vibe input acceleration (g's rms)

W_o = Power Spectral Density, PSD (g^2/Hz)

f_a, f_b = frequency limits (Hz)

The new narrow-band response is:

$z = y$

$$y_{rms} = \frac{9.8\,\ddot{y}_{rms}}{f_n^2} \quad \text{(inches rms)} \tag{86}$$

$$\ddot{y}_{rms} = \sqrt{\frac{\pi}{2}\, f_n\, W_o\, Q} \quad \text{(g's rms)} \tag{87}$$

$$\frac{y}{x} = \frac{\ddot{y}}{\ddot{x}} = Q \qquad \text{at resonance}$$

$$y_{rms} = \frac{12.28}{f_n^{1.5}} \sqrt{W_o\, Q} \quad \text{(inches rms)} \tag{88}$$

$$\sigma = \text{rms stress} = C_3\, y_{rms} \tag{89}$$

C_3 = configuration constant (KSI RMS/INCH RMS)

$$\sigma = C_3 \sqrt{\frac{\pi}{2}\, f_n Q}\quad W_o^{1/2} \qquad \text{(KSI RMS)} \tag{90}$$

It should be noted that σ is directly related to the value of W_o in the vicinity of the resonant frequency f_n (i.e. between the half-power points of the response curve). σ is only indirectly related to $\ddot{x}_{rms}$.

$$\ddot{x}_{rms} = \sqrt{W_o(f_b - f_a)} \tag{91}$$

The value of $\ddot{x}_{rms}$ can be changed by altering the value of W_o outside the vicinity of f_n. The magnitude of σ will not change significantly. Thus there is not a unique relationship between $\ddot{x}_{rms}$ and σ. There is between σ and the value of W_o in the vicinity of f_n.

For sine resonance dwell:

$$\frac{z}{x} = \frac{\ddot{y}}{\ddot{x}} = Q$$

$$S = \frac{\Delta S}{2} \quad \text{(KSI)}$$

S = stress amplitude

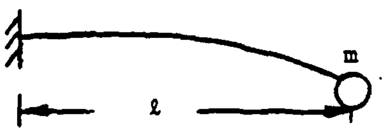
ΔS = peak-peak stress range

$\ddot{x}_s$ = sine vibe input acceleration (g's)

$S = C_1 z$

C_1 = configuration constant (KSI/INCH)

An example will be worked out to illustrate the use of the above expression. The system considered will be that of a massless beam of rectangular cross section with a concentrated mass load.



$$m = \frac{W}{g}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{m\ell^3}} \qquad (92)$$

W = weight (lbs)

m = mass (lb - sec^2/in)

g = acceleration of gravity = 386 in/sec^2

E = modules of elasticity (lb/in^2)

I = area moment of inertia of beam cross section (inches^4)

For a rectangular cross-section

$$I = \frac{1}{12} w h^3 \qquad (93)$$

$$c = h/2$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{3EI\ (386)}{w\ell^3}}$$

$$f_n = 5.42 \sqrt{\frac{EI}{w\ell^3}} \qquad (94)$$

Let w = 2 " ; h = 0.25"

Mat'l: 7075-T6

$E = 10.3 \times 10^6\ \mathrm{lb/in}^2$

$Q = 25$; $\beta = 9.65$

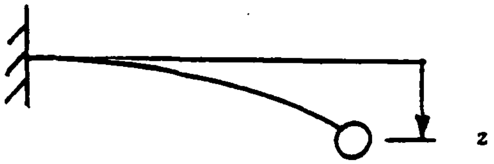
$\overline{C} = 80$ KSI ; $\overline{A} = 180$ KSI

W = 10 lbs.

$$I = \frac{1}{12} w h^3 = \frac{1}{12} (2)(1/4)^3 = 2.60 \times 10^{-3}\ \mathrm{in}^4$$

$\ell = 3.15$ inches

$$f_n = 5.42 \sqrt{\frac{(10.3 \times 10^6)(2.6 \times 10^{-3})}{10(3.15)^3}} = 50\mathrm{Hz}$$



$$z = \frac{P\ell^3}{3EI} \quad ; \quad P\ell = \frac{3EI}{\ell^2} z$$

$$S = \frac{(P\ell)(h/2)}{I} = \frac{3EI}{\ell^2} \times \frac{h}{2I} z = \frac{3Eh}{2\ell^2} z$$

$$\sigma = \frac{1.5Eh}{\ell^2} z_{rms}$$

$$S = \frac{1.5Eh}{\ell^2} z$$

$$S = C_1 z$$

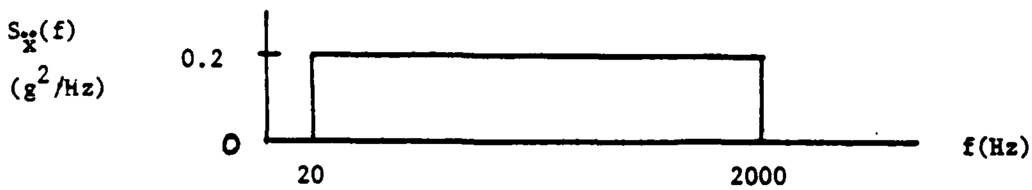
$$\sigma = C_3 y_{rms}$$

$$C_1 = \frac{1.5Eh}{\ell^2} \quad \text{(KSI/INCH)}$$

$$C_3 = \frac{1.5Eh}{\ell^2} \quad \text{(KSI RMS/INCH RMS)}$$

for E ~ KSI

$$C_1 = C_3 = \frac{(1.5)(10.3 \times 10^3)(0.25)}{(3.15)^2} = 389 \ \frac{\text{KSI}}{\text{INCH}}$$



$W_o = 0.2\ g^2/Hz$

$\ddot{x}_{rms} = 19.9\ g\ rms$

$\ddot{y}_{rms} = 19.8\ g\ rms$

$y_{rms} = 0.0777$ inches rms

$\sigma = C_3\, y_{rms} = (389)(0.0777) = 30.2$ KSI rms

Also

$$y_{rms} = \frac{12.28\ Q^{1/2}}{f_n^{1.5}(f_b - f_a)^{1/2}}\ \ddot{x}_{rms}$$

Define

$$C4 = \frac{C1\ (12.28)Q^{1/2}}{f_n^{1.5}(f_b - f_a)^{1/2}} \qquad \frac{\text{KSI RMS}}{\text{g RMS}}$$

$C4 = 1.517$; $\quad C1 = \dfrac{1.5Eh}{\ell^2}$

$\sigma = 1.517\ \ddot{x}_{rms}$ (KSI RMS)

For the sine resonance dwell:

$$\ddot{x}_s = \frac{x_s\ f_n^2}{9.8}\ g's$$

$$S = C_1\ z \approx C_1Qx = \frac{C_1Q\ 9.8\ \ddot{x}_s}{f_n^2}$$

Define

$$C_2 = \frac{C_1 \, Q \, 9.8}{f_n^2} = 38.1 \ \frac{KSI}{g}$$

$$S = C_2 \ddot{x}_s$$

$$S = 38.1 \ \ddot{x}_s$$

For $\ddot{x}_s = 1g$; $S = 38.1$ KSI

In summary for the example given where $W_o = 0.2 \ g^2/Hz$; $\ddot{x}_s = 1g$

$\sigma = 30.2$ KSI RMS

$S = 38.1$ KSI vector

From reference [2]

$$N_R = \left(\frac{\bar{C}}{\sigma}\right)^{\beta} = \left(\frac{80}{30.2}\right)^{9.65} = 1.15 \times 10^4 \text{ cycles} \qquad (95)$$

$$N_S = \left(\frac{\bar{A}}{S}\right)^{\beta} = \left(\frac{180}{38.1}\right)^{9.65} = 3.22 \times 10^6 \text{ cycles} \qquad (96)$$

where N_R and N_S are cycles to failure during the random and sine tests respectively.

$$N_R \neq N_S$$

Thus the value of $\ddot{x}_s$ and W_o chosen for the example are not equivalent.

The general equivalence case will be continued as follows:

$$S = \frac{\Delta S}{2} = \bar{A} \; N_S^{-1/\beta} \qquad C_1 = \frac{S}{z} = C_3 = \frac{\sigma}{z_{rms}}$$

$$S = C_2 \, \ddot{x}_s^{\,\eta_s}$$

$$N_S = f_n \; T_s \qquad C_4 = \frac{C_1 \; (12.28) \; Q^{1/2}}{f_n^{\,3/2} (f_b - f_a)^{1/2}}$$

$$\sigma = \bar{C} \, N_R^{-1/\beta}$$

$$\sigma = C_4 \, \ddot{x}_R^{\,\eta_R} \qquad C_2 = \frac{C_1 \; Q \; 9.8}{f_n^{\,2}}$$

$$N_R = f_n \; T_R$$

$$\left.\begin{aligned} \eta_S &= 0.714 \\ \eta_R &= 0.833 \end{aligned}\right\} \text{non-linear damping parameter for internal stress-strain hysteresis damping}$$

For the sine and random tests to be of the same duration, $\underline{T_S = T_R}$

or $\quad N_S = N_R$

$$N_S^{-1/\beta} = N_R^{-1/\beta}$$

$$\frac{S}{\bar{A}} = \frac{\sigma}{\bar{C}} \quad ; \quad \boxed{\frac{S}{\sigma} = \frac{\bar{A}}{\bar{C}}} \qquad\qquad \frac{C_2}{C_4} = \left(\frac{1}{1.25}\right)\sqrt{\frac{Q}{f_n}}\left(\frac{1}{f_b - f_a}\right)^{1/2}$$

$$\boxed{\frac{C_2\ \ddot{x}_S^{\ \eta_S}}{C_4\ \ddot{x}_R^{\ \eta_R}} = \frac{\bar{A}}{\bar{C}}} \qquad\qquad \ddot{x}_R^{\ \eta_R} = \left[W_o\ (f_b - f_a)\right]^{\eta_R/2}$$

Using previously defined expressions it can be shown that the desired equivalency expression is

$$\boxed{\ddot{x}_S^{\ \eta_S} = (1.25)\left(\frac{\bar{A}}{\bar{C}}\right)\sqrt{\frac{f_n}{Q}}\left[\frac{1}{f_b - f_a}\right]^{\frac{1-\eta_R}{2}} W_o^{\ \eta_R/2}} \tag{97}$$

The above expression is a function of η_S and η_R. In general η_S and η_R values are not restricted to any particular value. Thus from that standpoint it can be concluded that for structures with non-linear damping there is no single, unigue relationship between $\ddot{x}_S$ and W_o.

Consider the linear damping case where $\eta_S = \eta_R = 1$. The equivalency equation becomes:

$$\frac{\ddot{x}_S}{W_o^{1/2}} = (1.25)\left(\frac{\bar{A}}{\bar{C}}\right)\sqrt{\frac{f_n}{Q}} \tag{98}$$

From [1]

$$\frac{\bar{A}}{\bar{C}} = \sqrt{2}\left[\Gamma\left(\frac{2+\beta}{2}\right)\right]^{1/\beta} \tag{99}$$

where β = slope parameter of the material's sinusoidal fatigue curve

$\frac{\bar{A}}{\bar{C}}$ takes on values of about 2 for ductile materials and about 3 for brittle materials.

TABLE IX EQUIVALENCY RATIO VALUES

MATERIALS	$\ddot{x}_S/W_o{}^{1/2}$
7075-T6 (ductile)	2.81
AZ31-B (brittle)	3.98

$$\frac{3.98}{2.81} = 1.42$$

Thus it can be concluded that, if a "black box" contains ductile and brittle materials, there is no single, unique $\ddot{x}_S/W_o$ relationship.

It can also be seen that the equivalency expression is a function of f_n/Q. In general f_n/Q is not a constant. This fact also illustrates a lack of a unique equivalency relationship.

From Fracture Mechanics Considerations [7]

$\bar{A}_i$ = sinusoidal fatigue curve "y - intercept" parameter with an initial flaw of length a_i

$\bar{C}_i$ = random fatigue curve "y - intercept" parameter with an initial flaw of length a_i

$$\frac{\overline{A}_i}{\overline{C}_i} = \frac{\overline{A}}{\overline{C}} \quad ; \quad \begin{matrix} \overline{A}_i \neq \overline{A} \\ \overline{C}_i \neq \overline{C} \end{matrix}$$

Initial flaws do not alter the $\ddot{x}_S/W_o$ relationship.

For 63 - 37 Tin-Lead Solder in the high cycle fatigue region at room temperature:

Reversed bending:

$\overline{A} = 15.3$ KSI ; $\overline{C} = 6.74$ KSI

$\beta = 9.85$; $\overline{A}/\overline{C} = 2.27$

Reversed shear:

$\overline{A} = 14.5$ KSI ; $\overline{C} = 6.62$ KSI

$\beta = 8.97$; $\overline{A}/\overline{C} = 2.19$

For Copper Wire:

$\overline{A} = 81.9$ KSI ; $\overline{C} = 36.9$ KSI

$\beta = 9.28$; $\overline{A}/\overline{C} = 2.22$

The above solder and copper can be considered as ductile. If workmanship defects are treated as having initial flaws of some abitrary lengths, then

$$\overline{A}_i < \overline{A} \quad \text{and} \quad \overline{C}_i < \overline{C} .$$

However $\frac{\bar{A}_i}{\bar{C}_i} = \frac{\bar{A}}{\bar{C}} = 2.22$

The equivalency equation becomes

$$\frac{\ddot{x}_S}{W_o^{1/2}} = 2.78 \sqrt{\frac{f_n}{Q}} \quad (100)$$

In general $\frac{f_n}{Q} \neq$ constant. The structural damping (hence, Q) is composed of several damping mechanisms (e.g. friction, internal stress-strain hysteresis, air, viscoelastic). A single equivalence still does not exist.

$Q = 20$ is a typical value for an MLB without any special added viscoelastic damping treatment for f_n's ranging from 100 - 300 Hz.

For $W_o = 0.1\ g^2/Hz$

$$\ddot{x}_S = 0.197 \sqrt{f_n} \quad g\text{'s} \quad \text{(See Figure 13)} \quad (101)$$

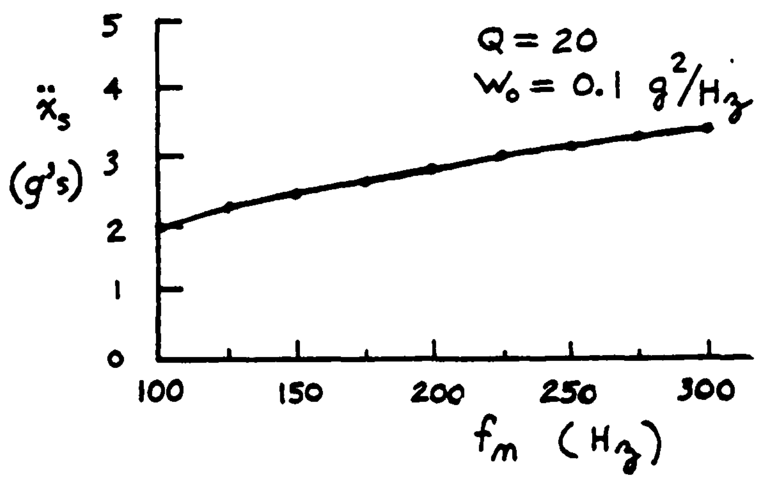


FIGURE 13 EQUIVALENT $\ddot{x}_S$

APPENDIX F

DERIVATION OF ACCELERATED RANDOM VIBE TEST ACCELERATION LEVEL

Refer to Appendix E for the definitions and units of the stress terms σ_1 , σ_2 , ΔS_1 , and ΔS_2 , the acceleration terms $\ddot{x}_1$ and $\ddot{x}_2$ and the stress per rms acceleration term C_4 .

For a given structural assembly where the shape of the input power spectral density $S_{\ddot{x}}(f)$ and the structural resonant frequency is the same at both levels 1 and 2:

$$\sigma_1 = C_4 \ddot{x}_1^{\eta} \quad ; \quad \sigma_2 = C_4 \ddot{x}_2^{\eta}$$

The equivalent sine stress levels are:

$$\Delta S_1 = \left(\frac{2\bar{A}}{\bar{\bar{C}}}\right) \sigma_1 = C_5 \ddot{x}_1^{\eta}$$

$$\Delta S_2 = \left(\frac{2\bar{A}}{\bar{\bar{C}}}\right) \sigma_2 = C_5 \ddot{x}_2^{\eta}$$

where $C5 = \left(\frac{2\bar{A}}{\bar{\bar{C}}}\right) C4$

$$N_{f_1} = \frac{2}{(\theta - 2)\ C_o\ C_5^{\theta}\ \ddot{x}_1^{(\theta + \eta)}\ Y^{\theta}} \left[\left(\frac{1}{a_i}\right)^{\frac{\theta - 2}{2}} - \left(\frac{1}{a_{c_1}}\right)^{\frac{\theta - 2}{2}} \right] \qquad (102)$$

$$N_{f_2} = \frac{2}{(\theta - 2)\ C_o\ C_5^{\theta}\ \ddot{x}_2^{(\theta + \eta)}\ Y^{\theta}} \left[\left(\frac{1}{a_i}\right)^{\frac{\theta - 2}{2}} - \left(\frac{1}{a_{c_2}}\right)^{\frac{\theta - 2}{2}} \right] \qquad (103)$$

$$a_{c_1} = \left[\frac{\Delta K_c}{Y\ C_5\ \ddot{x}_1^{\ \eta}}\right]^2 \quad ; \quad a_{c_2} = \left[\frac{\Delta K_c}{Y\ C_5\ \ddot{x}_2^{\ \eta}}\right]^2 \tag{104}$$

For equal damage $\frac{N_1}{N_{f_1}} = \frac{N_2}{N_{f_2}}$

$$\frac{N_2}{N_1} = \frac{N_{f_2}}{N_{f_1}} \quad ; \quad \frac{T_2}{T_1} = \frac{N_2}{N_1}$$

$$\frac{T_2}{T_1} = \left(\frac{\ddot{x}_1}{\ddot{x}_2}\right)^{\eta\theta} \left(\frac{1}{X}\right) \tag{105}$$

$$\left(\frac{1}{X}\right) = \left[\frac{1 - \left(\frac{a_i\ C_5^{\ 2}\ Y^2\ \ddot{x}_2^{\ 2\eta}}{\Delta K_c^{\ 2}}\right)^{\frac{\theta - 2}{2}}}{1 - \left(\frac{a_i\ C_5^{\ 2}\ Y^2\ \ddot{x}_1^{\ 2\eta}}{\Delta K_c^{\ 2}}\right)^{\frac{\theta - 2}{2}}}\right] \tag{106}$$

Define

$$h_5 = \left(\frac{a_i\ C_5^{\ 2}\ Y^2}{\Delta K_c^{\ 2}}\right)^{\frac{\theta - 2}{2}} \tag{107}$$

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$$L_5 = 1 - h_5\, \ddot{x}_1^{\,2\eta\left(\frac{\theta - 2}{2}\right)} \tag{108}$$

$$\left(\frac{1}{X}\right) = \frac{1}{L_5} - \frac{h_5}{L_5}\, \ddot{x}_2^{\,2\eta\left(\frac{\theta - 2}{2}\right)}$$

$$\ddot{x}_2 - \underbrace{\ddot{x}_1 \left(\frac{T_1}{T_2}\right)^{\frac{1}{\eta\theta}}}_{V5} \left(\frac{1}{X}\right)^{\frac{1}{\eta\theta}} = 0 \tag{109}$$

NOTE: X is a function of both $\ddot{x}_1$ and $\ddot{x}_2$

X will be negative if the selected value for $\ddot{x}_2$ is larger than

$$\ddot{x}_{2_{max}} = \left[\frac{\Delta K_c^{\,2}}{a_1\, C_5^{\,2}\, Y^2}\right]^{\frac{1}{2\eta}} \tag{110}$$

It can be seen that the damping term η alters the effective value of the crack growth rate parameter θ . That is, $\eta\theta$ will be greater or less than θ if η is greater or less than unity.

APPENDIX G

SUMMARY OF DEVELOPED EQUATIONS

FATIGUE ($a_i = 0$)

$$\frac{\ddot{x}_2}{\ddot{x}_1} = \left(\frac{T_1}{T_2}\right)^{1/\eta\beta} \quad \text{(SINE OR RANDOM)} \qquad (111)$$

$$\frac{S_2}{S_1} = \left(\frac{N_1}{N_2}\right)^{1/\beta} = \left(\frac{T_1}{T_2}\right)^{1/\beta} \quad \text{(SINE)} \qquad (112)$$

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{N_1}{N_2}\right)^{1/\beta} = \left(\frac{T_1}{T_2}\right)^{1/\beta} \quad \text{(RANDOM)} \qquad (113)$$

$$N = fT \qquad (114)$$

$$\frac{\Delta\varepsilon_2}{\Delta\varepsilon_1} = \left(\frac{N_1}{N_2}\right)^{1/\beta} \quad \text{(CYCLIC)} \qquad (115)$$

FRACTURE MECHANICS ($a_i > 0$)

$$\Delta S_2 - \Delta S_1 \left(\frac{N_1}{N_2}\right)^{1/\theta} \left(\frac{1}{X}\right)^{1/\theta} = 0 \qquad (116)$$

$$X = \left[\frac{1 - \left(\frac{a_i \; Y^2 \; \Delta S_1{}^2}{\Delta K_c{}^2} \right)^{\frac{\theta - 2}{2}}}{1 - \left(\frac{a_i \; Y^2 \; \Delta S_2{}^2}{\Delta K_c{}^2} \right)^{\frac{\theta - 2}{2}}} \right] \tag{117}$$

$$\ddot{x}_2 - \ddot{x}_1 \left(\frac{T_1}{T_2} \right)^{\frac{1}{\eta\theta}} \left(\frac{1}{X} \right)^{\frac{1}{\eta\theta}} = 0 \tag{118}$$

$$X = \left[\frac{1 - \left(\frac{a_i \; C_5{}^2 \; Y^2 \; \ddot{x}_1{}^{2\eta}}{\Delta K_c{}^2} \right)^{\frac{\theta - 2}{2}}}{1 - \left(\frac{a_i \; C_5{}^2 \; Y^2 \; \ddot{x}_2{}^{2\eta}}{\Delta K_c{}^2} \right)^{\frac{\theta - 2}{2}}} \right] \tag{119}$$

$$C5 = \left(\frac{2\bar{A}}{\bar{C}} \right) C4 \quad (120) \quad ; \quad \sigma = C4 \; \ddot{x}^{\eta} \tag{121}$$

SINE-RANDOM EQUIVALENCY

$$\ddot{x}_S{}^{\eta_s} = (1.25) \left(\frac{\bar{A}}{\bar{C}} \right) \sqrt{\frac{f_n}{Q}} \left[\frac{1}{f_b - f_a} \right]^{\frac{1 - \eta_R}{2}} W_o{}^{\eta_R/2} \tag{122}$$

$$\frac{\bar{A}}{\bar{C}} = \sqrt{2} \left[\Gamma \left(\frac{2 + \beta}{2} \right) \right]^{1/\beta} \tag{123}$$

APPENDIX H

TIME BETWEEN STRESS PEAKS

All of the high level random stress peaks may not occur in a test of finite duration. The primary reason is that such high peaks are low probability of occurrence events and don't occur very often. This section develops the approximate relationship between the average time between occurrences $\overline{T}$ of stress peaks L. L will be expressed in multiples α of the rms stress level σ. (i.e. $\alpha = L/\sigma$).

The envelope of positive stress has a Rayleigh probability density function. For values of stress greater than about 2.5σ the stress envelope approximates the stress peak values. The percent of time (i.e. probability) that the stress envelope is equal to or greater than α can be expressed as

$$p = e^{-\alpha^2/2} \tag{124}$$

$$\alpha = L/\sigma \tag{125}$$

For a random stress process of bandwidth B (Hz) the average time between independent events is approximated as

$$t_i \approx \frac{1}{B} \text{ seconds} \tag{126}$$

The number of independent opportunities for occurrence is

$$N_{opp} = \frac{1}{p} \tag{127}$$

$$\overline{T} = t_i N_{opp} = \frac{t_i}{p} = \frac{e^{\alpha^2/2}}{B}$$

$$\overline{T} = \frac{e^{\alpha^2/2}}{B} \text{ seconds} \tag{128}$$

The following conversion factors can be used with $\bar{T}$:

60 seconds/minute
3600 seconds/hour
720 hours/month
8640 hours/year

For $B = 500$ Hz :

α	$\bar{T}$
3	0.18 sec
3.5	0.91 sec
4	6.0 sec
4.5	50.0 sec
5	9.0 minutes
5.5	2.0 hours
6	36 hours (1.5 days)
6.5	1.2 months
7	2.8 years

Equation (128) can be rearranged as follows:

$$\alpha = \sqrt{2 \ln \bar{T} + 2 \ln B} \tag{129}$$

For $B = 500$ Hz and $\bar{T}$ in minutes

$$\alpha = \sqrt{20.618 + 2 \ln \bar{T}} \tag{130}$$

$\bar{T}$ (min)	α
5	4.88
10	5.02
15	5.10
20	5.16
30	5.24
60	5.37
120	5.5

APPENDIX I

SUMMARY REPORT

This appendix summarizes the more often used sections of the entire study. The equation and figure numbers are independent of those in the main body of the report.

It is typically of great interest and practical importance to accelerate a Gaussian random vibration qualification or acceptance test of an electronic "black box" in the laboratory from the actual service conditions. The test duration is compressed by a relatively large factor (e.g. 1000) with an attendant increase in the applied vibration level.

Mathematical relationships have been developed [1] which determine the proper increase in the vibration input root-mean-square (rms) level to the electronic "black box" for the desired test duration (i.e. time) compression factor such that the accumulated fatigue damage is the same for both the test and service environments. Cumulative fatigue damage does not necessarily mean structural fracture or failure. It means that useful life is being consumed and indicates the potential for failure. The failure potential must be the same at both environments.

This paper shows the criteria for selecting the form of the input vibration level-duration relationship and assigning values to the parameters. Derivations are shown in reference [1].

CONDITIONS OF SIMILITUDE

Certain conditions of similitude must be imposed upon the service and laboratory accelerated test environments if the developed mathematical relationships are to be appropriately and accurately applied. The fundamental hypothesis is that the damage states and damage rates must be the same for both environments. Specifically the states of stress (torsion, bending, axial), the corresponding fatigue strengths, the resonant mode shapes, the internal response stress spectrum shapes, the stress peak distribution, and the type and location of failure mechanisms must be the same for both environments.

Extreme temperature, humidity or corrosive element differences between the service and test environments may result in similitude violations, if such differences are sufficient to alter the material's fatigue strength parameters. Threshold sensitive or other non-linear response effects in general tend to violate conditions of similitude. In some cases lack of similitude can be quantitatively compensated for. Several examples are included.

The condition that the shape of the vibration acceleration input spectra or the overall acceleration rms levels must be the same for both environments has purposely been omitted from the previously listed conditions. This is because the fatigue damage state and rate are only indirectly related to the input acceleration spectrum. They are directly related to the response stress spectrum at the location where damage is accumulating. The response stress spectrum is related to the vibration acceleration power spectral density value in the vicinity of resonances.

CUMULATIVE FATIGUE

Black box structural elements (e.g. solder joints, wires, device leads, support structures) that are subjected to random vibration loads will always cumulate fatigue damage. Such fatigue damage can range in value from very little to very large. It is never zero. Structure element fracture occurs when the cumulated fatigue damage becomes large and is defined mathematically by the material's fatigue curve.

It can be shown that curves of equal damage have the same slope as the material's fatigue curve. Fatigue curve parameter values are readily available from many published sources (e.g. MIL-HDBK-5C, SAE J1099). Therefore it is useful to work with fatigue curve parameters for determining test acceleration factors whether or not large fatigue damage is accumulated.

For sinusoidal stressing the material's fatigue curve is expressed as:

$$S = \bar{A} N_s^{-1/\beta} \tag{1}$$

where S = Stress amplitude (ksi)

N_s = number of sinusoidal stress cycles to failure

$\bar{A}$ = y-intercept on log - log plot for $N_s = 1$; true ultimate stress (ksi)

β = slope parameter

For Gaussian random stressing the material's fatigue curve is expressed as [2] :

$$\sigma = \bar{C} N_m^{-1/\beta} \tag{2}$$

where σ = rms stress value (ksi)

N_m = median cycles to failure

$\overline{C}$ = $\overline{y}$-intercept on log - log plot (ksi)

β = slope parameter (see Table I)

$$\overline{C} = \left[\frac{\overline{A}}{\sqrt{2}}\right]\left[\frac{1}{\Gamma\left(\frac{2+\beta}{2}\right)}\right]^{1/\beta} \tag{3}$$

The random fatigue curve parameters can be obtained from the sinusoidal fatigue curve. It should be noted that $\beta \approx 9$ for ductile materials and $\beta \approx 20$ for brittle materials regardless of the material's ultimate strength.

Equation (2) shows that the fatigue process is directly related to the rms stress level σ of the structural element (e.g. solder joint) inside the electronic black box being stressed and the median number of applied stress cycles N_m . Both σ and N_m are most frequently estimated by determining the resonant response of all the structural members of the black box and adjacent structures.

TABLE I TYPICAL β VALUES

MATERIAL	β
Copper Wire	9.28
Aluminum Alloy:	
6061-T6	8.92
7075-T6	9.65
Soft Solder (63-37 Tin-Lead)	9.85
4340 (BHN 243)	10.5
4340 (BHN 350)	13.2
AZ31B Magnesium Alloy	22.4

FATIGUE-TEST RELATIONSHIPS

It was previously shown that the fatigue process was directly related to the rms stress level σ and the median number of applied stress cycles N_m. For purposes of test it is of more interest to express cumulative damage in terms of vibration input acceleration rms level $\ddot{x}$ to the electronic black box and test duration T. σ can be related to $\ddot{x}$ as follows:

$$\sigma = C_4 \ddot{x}^{\eta} \tag{4}$$

C_4 = constant (ksi/g rms)

η = damping parameter

Table II shows typical η values. Figure 1 shows typical σ-$\ddot{x}$ relationships for various η values with C_4 = 1.0 ksi/g. The value of η is best determined empirically because actual black boxes are composed of a mixture of varied damping types.

TABLE II TYPICAL η VALUES

DAMPING TYPE	η
LINEAR	1.0
INTERNAL HYSTERESIS	0.833
NON-LINEAR SPRING (INCREASING STIFFNESS)	≈1.2
COULOMB FRICTION	1.5

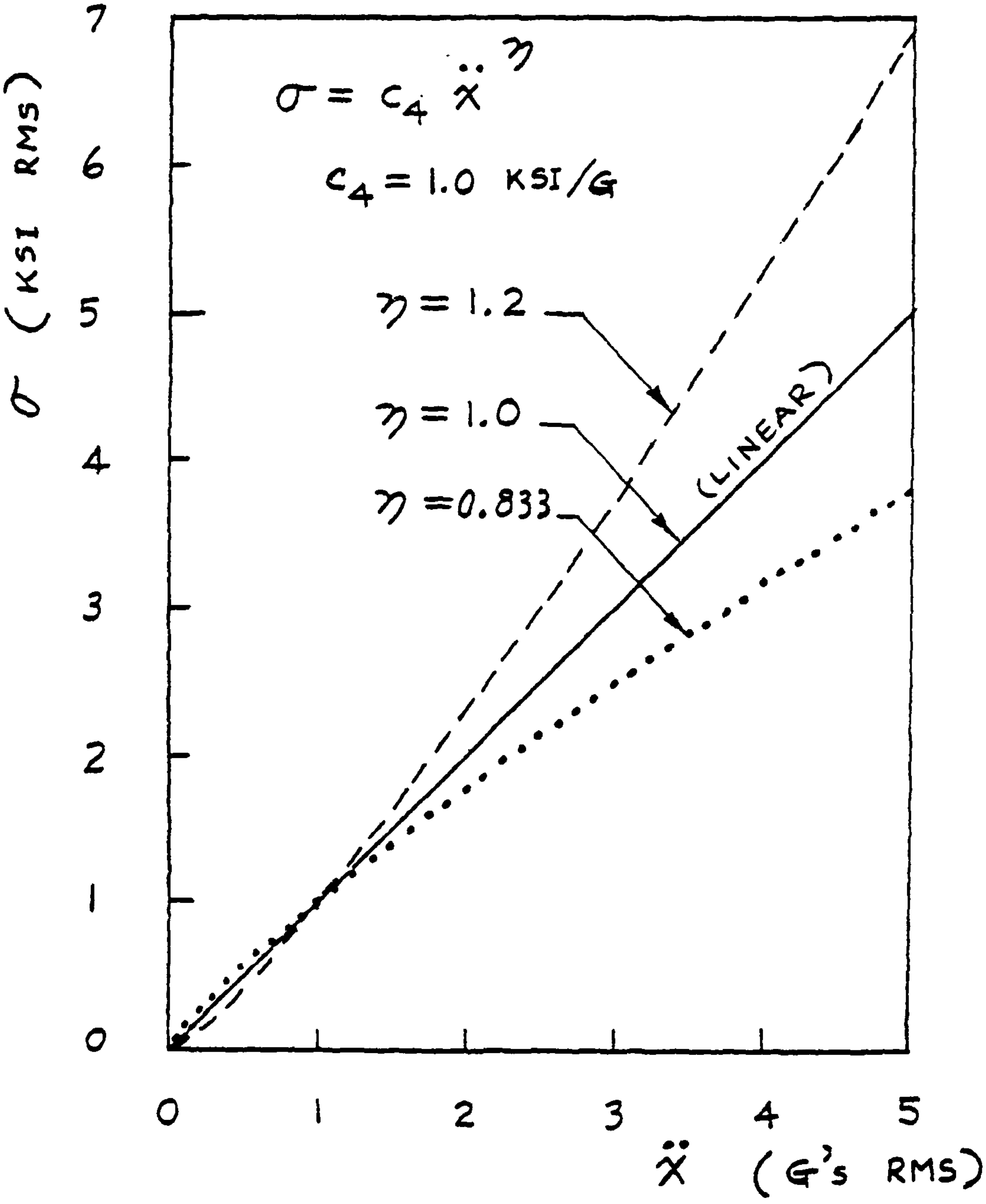

FIGURE 1 TYPICAL σ-$\ddot{x}$ RELATIONSHIPS

Table II shows that $\eta = 0.833$ for internal hysteresis. Consider the case of a solder joint on a multi-layer board (MLB). There exists some amount of internal hysteresis damping due to the solder joint stressing. However, it is more likely that the relationship between solder joint stress σ and $\ddot{x}$ of equation (4) is governed by the friction damping at the MLB support edges or by some special viscoelastic MLB damping treatment. Therefore, for this case in general $\eta \neq 0.833$.

The median stress cycles N_m can be related to test duration T as follows:

$$N_m = f_{eff}\, T \tag{5}$$

f_{eff} = effective frequency (Hz) = rate of zero crossings

For a single-degree-of-freedom system [2]

$$f_{eff} = f_o = \text{center frequency of response spectrum} = \text{resonant frequency} \tag{6}$$

For a two-degree-of-freedom system (2DF) [3]

$$f_{eff} = \sqrt{\frac{\sigma_a^2}{\sigma_T^2}\, f_a^2 + \frac{\sigma_b^2}{\sigma_T^2}\, f_b^2} \tag{7}$$

f_a = 1st mode resonant frequency (Hz)

f_b = 2nd mode resonant frequency (Hz)

σ_a = 1st mode rms stress level (ksi)

σ_b = 2nd mode rms stress level (ksi)

σ_T = total rms stress level (ksi)

$$f_a < f_{eff} < f_b$$

$$\sigma_T = \sqrt{\sigma_a^2 + \sigma_b^2} \tag{8}$$

The two-degree-of-freedom system will later be used in an example.

ACCELERATION FACTOR EXPRESSIONS

Two separate acceleration factor expressions will be presented. The first expression is for usually thought of fatigue case where any existing initial cracks (i.e. flaws) are not considered. The second expression is the Fracture Mechanics case where initial cracks, either actual or postulated, are considered to exist. Both expressions are of practical importance. The service environment parameters will be denoted by the subscript 1. The accelerated test environment parameters will be denoted by the subscript 2.

NO INITIAL CRACKS

Reference [1] shows that the appropriate acceleration factor expression is as follows:

$$\ddot{x}_2 = \ddot{x}_1 \left(\frac{T_1}{T_2}\right)^{1/\eta\beta} \quad \text{(g rms)} \tag{9}$$

<u>EXAMPLE 1</u>

Given: The service parameters

$\ddot{x}_1 = 1.0$ g rms; $\eta = 0.833$

$T_1 = 1000$ hours

The desired $T_2 = 1$ hour

Find: $\ddot{x}_2$ for a black box with copper wire as the critical structural element.

Solution: From Table I $\beta = 9.28$

$$\ddot{x}_2 = 1.0 \left(\frac{1000}{1}\right)^{1/7.73} = 2.44 \text{ g rms}$$

DUCTILITY EFFECT

Equation (9) can be used to show the effect of the material's ductility. Ductility is the ability of the material to be deformed without fracturing. Ductile materials have values of $\beta \approx 9$. Brittle materials have values of $\beta \approx 22$. See Table I. Equation (9) can be plotted in a normalized fashion. Figure 2 shows a plot of acceleration for two diverse β values with $\eta = 0.833$. It can be seen that the acceleration factor is sensitive to β values (i.e. ductility); brittle materials being the most sensitive.

DAMPING LINEARITY EFFECT

Figure 3 is a plot of equation (9) for two η values with $\beta = 9.28$. For a time compression factor of 1000 the acceleration factor is 2.44 for $\eta = 0.833$ and 1.86 for $\eta = 1.2$. The acceleration factor is sensitive to η values; large η values being the most sensitive.

EQUAL DAMAGE

Equal cumulative fatigue damage exists at all points along any one line (i.e. curve) of figures 2 and 3. However, the damage is not equal from curve to curve, even at the point where all the curves intersect.

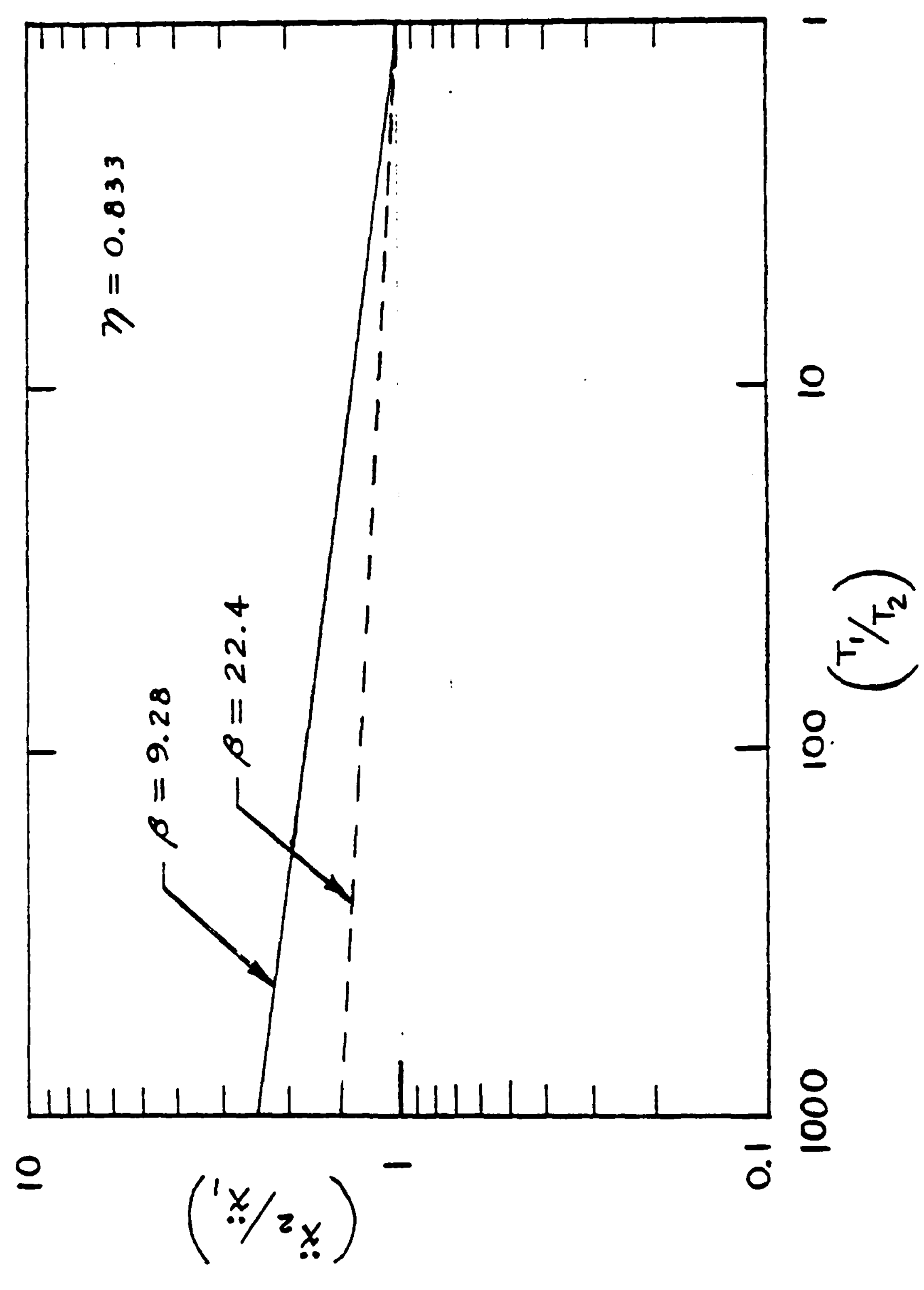

FIGURE 2 Acceleration Factor for Variable β

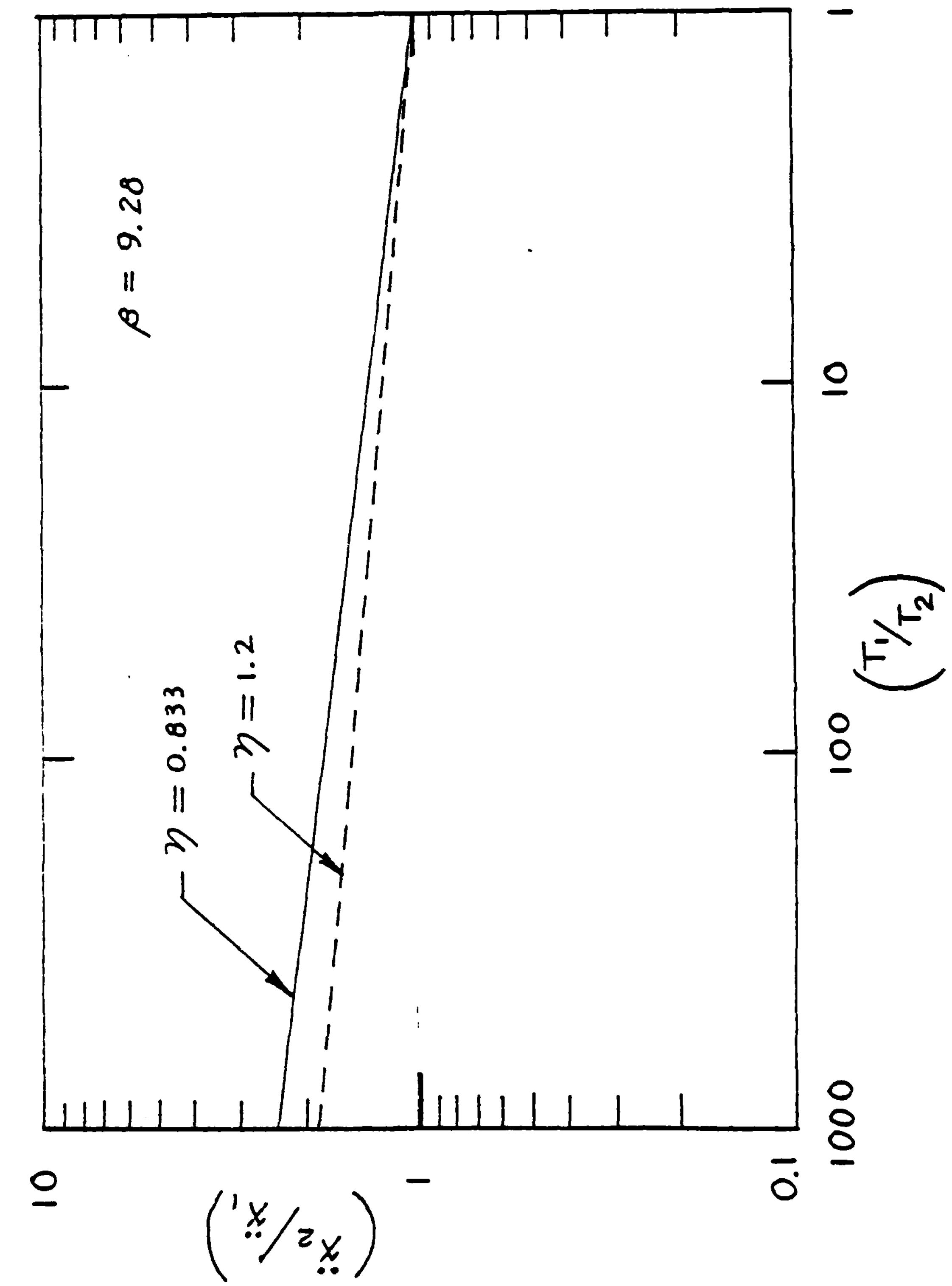

FIGURE 3 Acceleration Factor for Variable η

FRACTURE MECHANICS EFFECTS

Initial cracks (flaws) can exist in structural elements as a result of metallurgical inclusions, of fabrication or assembly procedures, or temporary overloads. This amounts to damage being cumulated prior to the service or test environments. The acceleration factor expression is as follows:

$$\ddot{x}_2 - \ddot{x}_1 \left(\frac{T_1}{T_2}\right)^{1/n\theta} \left(\frac{1}{X}\right)^{1/n\theta} = 0 \tag{10}$$

$$\frac{1}{X} = \frac{1 - \left(\dfrac{a_i\ C_5^{\ 2}\ Y^2\ \ddot{x}_2^{\ 2n}}{\Delta K_c^{\ 2}}\right)^{\frac{\theta - 2}{2}}}{1 - \left(\dfrac{a_i\ C_5^{\ 2}\ Y^2\ \ddot{x}_1^{\ 2n}}{\Delta K_c^{\ 2}}\right)^{\frac{\theta - 2}{2}}} \tag{11}$$

$$C_5 = \left(\frac{2\ \bar{A}}{\bar{\bar{C}}}\right) C_4 \qquad \text{(ksi/g rms)} \tag{12}$$

a_i = initial crack length (inches)

Y = geometrical parameter

ΔK_c = material's fracture toughness (ksi $\sqrt{\text{in}}$)

θ = slope parameter of material's crack growth rate curve.

Equation (10) is a transcendental equation that is most conveniently solved by a method developed in reference [1]. It cannot be normalized like equation (9) because of the inherent non-linearity of the Fracture Mechanics process.

Table III shows several typical θ values. By comparing Table III with Table I it can be seen that $\theta \approx \beta/2$. On a log - log plot the curve of equation (9) is $(1/\eta\beta)$ whereas the slope of equation (10) is $(1/\eta\theta)$. This fact makes the acceleration factor of equation (10) quite different from that obtained by equation (9) due to the mere existence of initial cracks. Compare Figure 4 of Example 2 below with Figures 2 and 3.

TABLE III TYPICAL θ VALUES

MATERIAL	θ
Cr-Mo-V	4.09
4340	4.65
7075-T6	4.00

The fatigue life of a structural element is greatly reduced by the existence of initial cracks. However, the acceleration factor obtained from equation (10) is not very sensitive to various a_i values. It is sensitive to η values.

EXAMPLE 2

Given: Critical element material: 7075-T6 Aluminum alloy

a_i = 0.007 inches

$\ddot{x}_1$ = 1.0 g rms; C_4 = 1.0 ksi/g rms

T_1 = 1000 hours

Find: $\ddot{x}_2$ as a function of T (0.5 hours $\leq T_2 \leq$ 1000 hours) for η = 0.833, 1, 1.2.

Solution: Use equations (10) - (12). The results are shown in figure 4. It can be seen that the acceleration factor is sensitive to the value of η. For T_2 = 1 hour $\ddot{x}_2$ = 8 g rms for η = 0.833. $\ddot{x}_2$ = 2.4 g rms in example 1 which has no initial crack. Thus Fracture Mechanics effects greatly influence the acceleration factor value. As previously mentioned in the EQUAL DAMAGE section the cumulative fatigue damage is the same along all points of a given curve but not between curves.

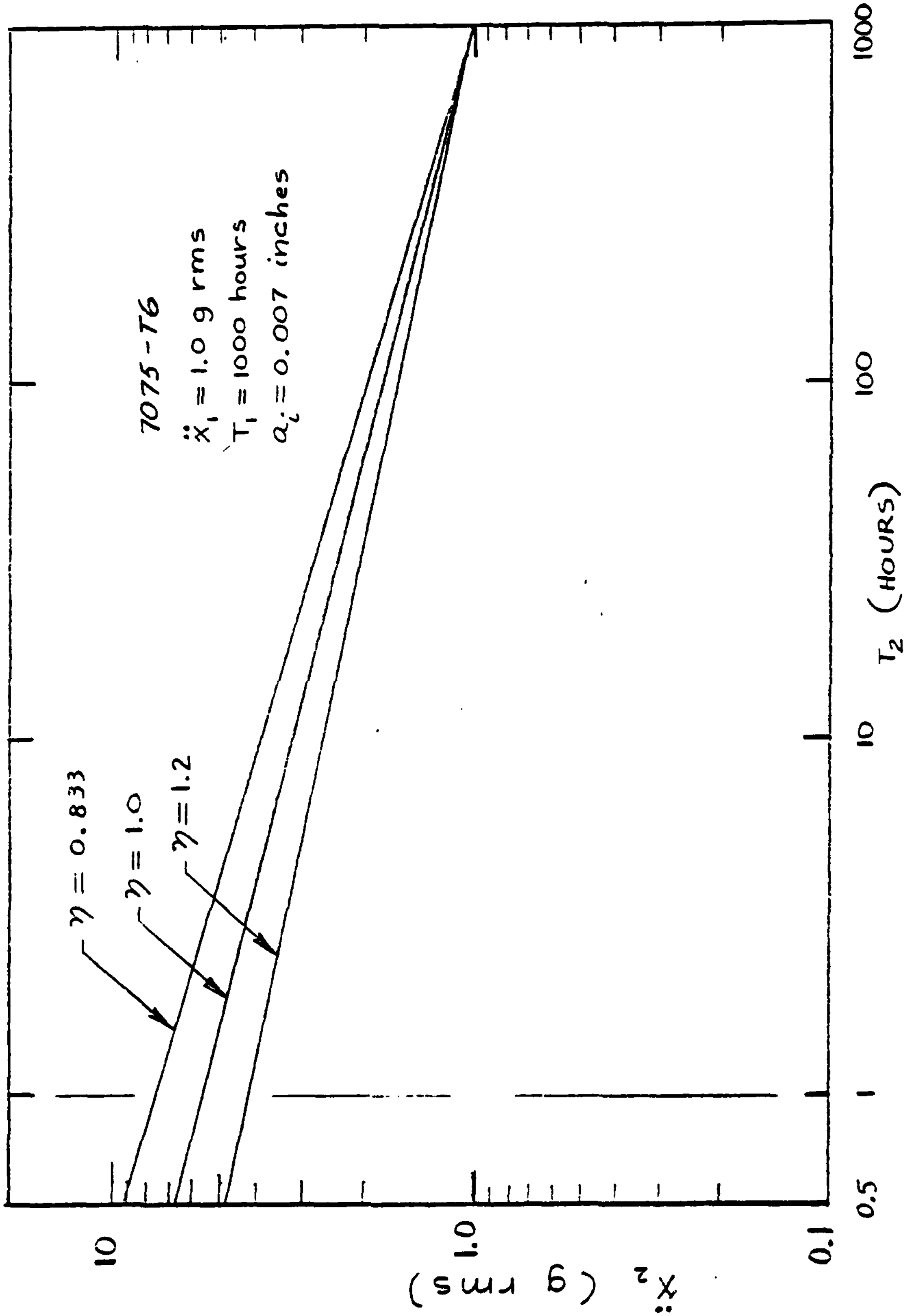

FIGURE 4 ACCELERATED RANDOM INPUT (η VARIABLES)

MULTI-FACTOR CONSIDERATIONS

In general all the structural elements in a relatively complex electronic black box will not have the same acceleration factor. Yet a single factor value must be chosen for the accelerated test. Such a selection is considered to be subjective. The value chosen will result in a proper test for only one class of structural elements. The other elements will be either under or over-tested. An average factor would give average results. The most conservative approach would be to select the largest acceleration factor value.

SIMILITUDE VIOLATION COMPENSATION

Some similitude violations can be compensated. One such example will be shown.

EXAMPLE 3

Given: The similitude violation is due to a difference in stress spectra between service and test environments. The stress vibration system is 2DF. The calculated acceleration factor ($\ddot{x}_2/\ddot{x}_1$) is 3 for the desired time compression factor (T_2/T_1) using equation (9). The spectra parameter values are given below:

PARAMETER	ENVIRONMENT	
	SERVICE	TEST
σ_a (ksi)	8	24
σ_b (ksi)	16	20
f_a (Hz)	150	150
f_b (Hz)	375	375
σ_T (ksi)	17.9	31.2
f_{eff} (Hz)	342	264

Find: the appropriate test compensation factors such that $\sigma_{T_{TEST}} = 3\,\sigma_{T_{SERVICE}}$ and $f_{eff_{TEST}} = f_{eff_{SERVICE}}$.

Solution: It can be seen that the resonant frequencies are the same at both environments. However, σ_b did not increase from 16 to 48 ksi as desired. This would cause an inappropriate test damage state and rate. $\sigma_{T_{TEST}}$ should be 3 x 17.9 = 53.7 ksi. Therefore, $\ddot{x}_2$

needs to be increased by an additional factor of 53.7/31.2 = 1.72. The test duration needs to be increased from its computed compressed value by a factor of 342/264 = 1.3.

EXAMPLE 4

Given: The similitude violation is due to a difference in the amplitude distribution of the stress peaks. The motion of the structural element being stressed will be snubbed (i.e. limited) if the motion exceeds a specified displacement. At the service vibration level motion limiting occurs such that the stress is limited at 5σ. At the accelerated test level the stress is limited at 3σ. The structural element being stressed is copper wire. $\ddot{x}_2$ was determined for the desired time compression factor (T_2/T_1) using equation (9). T_2 was computed to be 30 minutes.

Find: The modified time T_2 such that equal damage is done at both service and test levels.

Solution: Most fatigue damage during random vibration is caused by stress peaks between 2σ and 5σ. Reference [1] shows the following for copper wire:

1. Limiting stresses at 5σ is equivalent to no limiting.
2. Limiting stresses at 3σ will extend the fatigue life by a factor of 1.86.
3. 5σ peaks occur approximately every 9 minutes on the average.

5σ peaks would occur during the 30 minute test duration if motion were not limited. The compensation technique is to extend the test time by a factor of 1.86. Thus, T_2 = 1.86 x 30 = 56 minutes.